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A semiotic analysis of multiple systems of logic: using tagmemic theory to assess the usefulness and limitations of formal logics, and to produce a mathematical lattice model including multiple systems of logic

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Abstract: Tagmemic theory as a semiotic theory can be used to analyze multiple systems of logic and to assess their strengths and weaknesses. This analysis constitutes an application of semiotics and also a contribution to understanding of the nature of logic within the context of human meaning. Each system of logic is best adapted to represent one portion of human rationality. Acknowledging this correlation between systems and their targets helps explain the usefulness of more than one system. Among these systems, the two-valued system of classical logic takes its place. All the systems of logic can be incorporated into a complex mathematical model that has a place for each system and that represents a larger whole in human reasoning. The model can represent why tight formal systems of logic can be applied in some contexts with great success, but in other contexts are not directly applicable. The result suggests that human reasoning is innately richer than any one formal system of logic.

Keywords: context; formal logic; perspective; representation; tagmemic theory; variation

A semiotic analysis of systems of formal logic can offer a contribution to semiotics and also a contribution to the understanding of formal logic. It can do the latter because semiotics is useful in placing any one smaller area of scholarly analysis within the larger context of human sign systems and human meanings expressed in sign systems.

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1 Multiple systems of logic

The specific issue that we wish to study semiotically is how we evaluate the meaning of *multiple* systems of formal logic. Why are there multiple systems, and how should we think about them? Aristotle's (1938) logic is two-valued. That is to say, each proposition in his system of categorical logic is either true or false. But in the twentieth century we find systems that introduce a third value, "Unknown," or multiple values, in addition to true and false. We also have systems that deal with the concepts of necessity and possibility and the concept of moral obligation. What insights can a semiotic analysis of logic provide about these systems and about the larger question of the nature of human rationality?

The short answer is that each system of formal logic is adapted to the study of *selected* aspects within the larger realm of human rationality. For example, a system with "Unknown" as a specific third value, in addition to true and false, is adapted to representing situations with widespread uncertainty. The modal logic of necessity is adapted to situations where human beings are studying necessity.

2 Specific systems of formal logic

Let us briefly survey some of the main systems of "formal logic" or "symbolic logic." The classical logic of earlier centuries in the West, beginning with Aristotle (1938), is perhaps most effectively represented in modern times by the conventional formalisms of propositional logic, first-order quantification, and various second-order extensions to first-order quantification (Copi 1979; Shapiro and Kouri Kissel 2018). But standing alongside these systems are other systems with various properties. We have many-valued logics (Gottwald 2017), intuitionistic logics (Moschovakis 2018), fuzzy logic (Cintula et al. 2017), quantum logic (Wilce 2017a), and modal logics (Garson 2018). Which of these can unequivocally claim to represent perfectly the nature of true rationality?

3 The value of a theory of theories

Can we obtain some insight into the multiplicity of systems of formal logic by using a semiotically oriented theory of theories? Peirce (2011 [1897–1910]) considered logic in general as semiotic. But our focus is narrower, on formal systems of logic. Semiotics, as a general theory of signs, can consider any well-structured system of formal logic as a system of signs. Any one system of logic is not completely isolated from the larger context of human communication in which it is embedded. A system of logic is not an arbitrary collection of hen scratches in the dirt, but a system whose signs have meaning. And the meaning eventually derives from the larger context of personal communication. Human beings create new special symbols with special meanings. And they create whole systems of signs, such as the systems in symbolic logic. They explain the new signs or the new systems of signs using human language and perhaps other supplemental mathematical or logical symbols that are already in place at an earlier stage.

Semiotics can readily analyze *multiple* systems of logic, not just one. Each limited, formalized system of logic represents a kind of theory of reasoning, or at least a theory about some *part* of reasoning. Multiple logics offer us multiple theories. Semiotics can analyze each of these theories using its own theoretical semiotic framework. In this respect, semiotics can function, among other things, as a theory of theories (Poythress 2013a, 2015, 2021). Each system of logic can be placed as a distinct subsystem within the general framework of a single overarching semiotic theory. In addition, if the analyst stands back from previous work in semiotics, he can analyze a previous work about semiotics using semiotics. And then he is engaging in working on a theory of a theory of theories. This position is analogous to what has sometimes been called "metalinguistics." Metalinguistics is the linguistic study of the verbal discussions of linguists, who themselves may study not only verbal communication in general, but verbal communication among linguists.

In fact, we can picture an ascending hierarchy of theories (Poythress 2021: §6). There are first-order theories (such as systems of formal logic). There are second-order theories, or theories of theories, that study the first-order systems of logic. There are third-order theories, which study second-order theories and their perspectives on first-order theories. And so on up to fourth-order theories, fifth-order theories, in an endless hierarchy (Poythress 2021: §§6, 8).

Given the lack of closure of the hierarchy, this kind of situation is aptly studied by a theory like tagmemic theory, a semiotic theory that *explicitly* acknowledges the role of hierarchy and the role of human participants and human theory-makers (Pike 1976, 1982: 3, 10, 67–106; Poythress 2021: §2). The ascending hierarchy is part of the articulation of the theory, rather than being extraneous to it (Poythress 2021: §7).

Tagmemic theory is most commonly known as a *linguistic* theory, a theory for the analysis of ordinary verbal language. But in fact the principal architect of the theory, Pike (1967, 1982), explicitly developed it as a system useful in analyzing human behavior in general. The theory treats verbal communications along with other uses of nonverbal signs in their occurrences within a larger human context of meaning. Thus tagmemic theory is a form of semiotics. In addition, Pike's (1959, 1982: 5–6, 19–38) development of the concept of *perspective* enabled tagmemics to become a theory of theories. Each first-order theory offers a *perspective* on its subject-matter. Tagmemic theory uses multiple perspectives, enabling it to study and classify first-order theories. It is therefore suitable for analyzing multiple systems of logic. Each system of logic will be treated as a distinct first-order theory about its subject matter. Tagmemic theory functions as a second-order theory, a theory of theories, to analyze each system of logic in relation to its subject matter. The second-order theory offers us a means to stand *outside* both a particular system of logic and the subject-matter to which logical theorists apply the system.

In short, we use tagmemic theory because its has built-in ability to use two related approaches: (1) to function as a theory of theories; and (2) to study the theorists, that is, the logicians, as real people who communicate (Pike 1976, 1982: 5–9). The key is to study how logicians relate their theories to the spheres of reasoning that they are studying.

4 Two-valued logic

How then do we evaluate various systems of formal logic? Each particular form of logic is partly designed to offer a model of deductive reasoning with respect to a particular sphere. For many studies in logic, it is natural to start with the sphere of mathematics, because of its high level of rigor. Classical two-valued logic deals with propositions that have one of two values: true or false. It represents well the kind of reasoning that can take place in elementary arithmetic and elementary geometry. Logicians can then undertake to expand the scope of this starting system of logic by trying to apply it to broader spheres.

But we run into challenges. Ever since Gödel (1931, 2000) put forward his incompleteness proofs concerning arithmetic, it has been known that there are some truths about arithmetic that cannot be proved using the usual starting axioms (Raatikainen 2018; Rosser 1939). This situation introduces a new kind of proposition, a proposition that is true but unprovable.

As we have said, classical two-valued logic starts with a system in which propositions are either true or false. Technically, in a system with quantification ("all" and "there exists"), there are also propositions with "free variables," such as "$1 + x = 3$," where x is a free variable. These propositions are not true or false until we specify some particular value for x. But we can say that every proposition is either true or false once we eliminate all free variables.

But now, from the standpoint of provability, we can say that there are not two but four kinds of propositions in standard Peano arithmetic (Halbach and Leigh 2020: §1.2): (1) propositions that are provable and true; (2) propositions that are true but not provable; (3) propositions that are false and can be proved to be false; and (4) propositions that are false but cannot be proved false.

(An extra qualification is noteworthy. This fourfold classification depends on the assumption that standard arithmetic is *consistent*. But Gödel's [1931, 2000] work also showed that if it is consistent, its consistency cannot be proved using only the resources given to us in standard arithmetic.)

5 Many-valued logics

It gets even more challenging. There are also propositions, such as the Goldbach conjecture (Bridges and Palmgren 2015: §1), that may be provably true (category 1), but for which a proof has not yet been found (as of 2019). Partly for philosophical reasons, mathematical "intuitionists" (Iemhoff 2019) developed an approach that allowed for three categories: (1) propositions that have been proved true; (2) propositions that have been proved false; and (3) propositions for which we do not know whether they are true or false. So people developed more than one form of "intuitionistic logic" to try to represent in formal terms the kind of deductive reasoning that was allowed under such circumstances (Moschovakis 2018). The intuitionists were best known for not accepting that the existence of some particular mathematical object, such as a set or a natural number, could be deduced merely by *reductio ad absurdum* – showing that the assumption of nonexistence led to a contradiction. Rather, one had to be able actually to construct an example of a set or a number with the required properties.

When we try to apply systems of logic beyond the sphere of mathematics, we find additional complexities. Some propositions might not be either true or false. That might be so, according to the intuitionists, because we can never *know* whether they are true or false. Or it might be because there is something defective in the way the propositions are formed. A famous case is the proposition, "The present king of France is bald." The difficulty is that there is no longer a king of France. The expression "the present king of France" does not designate anyone. So it seems incoherent to say that he is bald. It is equally incoherent to say that he is not bald. So what do we say?

There are several possible remedies within systems of logic. They all involve producing something like a broad policy for dealing with propositions that fail to refer properly. One policy is to declare that such propositions belong to a *third* category, "defective," which exists alongside "true" and "false." Likewise, one strategy for intuitionists is to declare that some propositions belong to a third category. Instead of "T" for true and "F" for false, we can have "U" for unknown truth value. We then have what is called a “three-valued” logic, instead of a “two-valued” logic. The three values are T, F, and U.

And then other people in their ingenuity can try postulating even more truth values. In principle, we can have four-valued and five-valued logics (Gottwald 2017: §§3.1–3.2). Or we can have a continuum of values: any real number from 0 (representing falsehood) to 1 (representing truth). This situation with a continuum of values has been called “fuzzy logic” (Cintula et al. 2017). It is intended to represent mathematically a situation of “partial truth.”

By analyzing these systems of logic from *outside*, within a larger context, we can better see their value and their limitations. We recognize that the logicians are tailoring the system of logic to the sphere to which they want to apply it. We are then in a good position to affirm the potential value of *multiple* systems of logic. The systems do not necessarily have to be interpreted as in competition, but rather as tailored for different purposes on the part of the theorists who construct and use them. Using tagmemic theory, we may say that each system offers a kind of *perspective* on reason and rationality (Pike 1982: 5–6, 19–38). A single perspective may be insightful without being a total, masterful solution to the general understanding of human rationality. That general understanding includes acknowledgment of the hierarchy of theories, and as a consequence any formalized representation can never capture all of human thought (Poythress 2021: §8).

6 Extensions of logic

We may consider next how theorists have built *extensions* of classical logic to deal with new areas of study that are already exemplified in natural languages. One such extension is “modal logic” (Garson 2018). Modal logic exists in order to try to represent in a rigorous form issues involving necessity and possibility, rather than only with simple propositions that are true or false with respect to the actual world.

Consider an example. Napoleon Bonaparte died on 5 May 1821. That is a true proposition. But it is not *necessarily* true. We can picture a world in which Napoleon did not exist. Or we can picture a world where Napoleon died on 3 May 1821. We might say, “It is possible that Napoleon would not have existed,” and “It is possible that Napoleon could have died on 3 May 1821, or a few days earlier.” On the other hand, it would appear that a truth like “2 + 2 = 4” is *necessarily* true. So logicians have developed a symbolic notation and some extra axioms in order to represent in formal symbolism what is going on when we reason about necessity and possibility.

Logicians have also produced extensions that represent still other areas of natural language. There is, for example, the language of moral obligation, "deontic logic" (Garson 2018: §3). We *ought not* to murder people. Then there are forms of "modal" logic that attempt to represent, not what is necessary or possible in the world, but what goes on in human knowledge ("epistemic logic") or what goes on in reasoning about propositions that have tense and refer to events taking place at specific times ("temporal logic"). The modal logic of necessity is sometimes distinguished from these latter forms by labeling it "alethic logic" (Garson 2018: §2; McNamara 2019: §1.1).

7 Fuzzy categories

We may also consider how formal systems of logic struggle in dealing with the "fuzzy boundaries" in meanings in natural language. What does it mean for a painting to be "beautiful"? The word *beautiful* has a fuzzy boundary. That is to say, there is no exact point that we can map out, in the spectrum of all paintings, where suddenly the paintings beyond that point cease to be beautiful.

Admittedly, the idea of the beautiful may be difficult to pin down. But the problem exists in more prosaic cases.

What does it mean to say that the liquid in my glass is water? The chemist may define water as H_2O. But that is a technical definition. Ordinary usage differs. For the purposes of ordinary communication, when we say "water," we mean something that functions for practical purposes the way water functions. It looks like water, it tastes like water, it quenches thirst, and chemically it is mostly water. But it probably has minor quantities of dissolved salts, and some dissolved oxygen and nitrogen from the air. It may even be dirty water. Dirty water, after all, is something that we still call water! What if it has a tiny bit of fruit juice mixed in? Or if it is carbonated water? How much can it deviate from pure H_2O and still be "water" according to the standards of everyday communication? In ordinary language, the term "water" has a fuzzy boundary. There are liquids that clearly are water; there are other liquids (milk) that clearly are not. But in between there is at least some area of no-man's-land, where we may hesitate. We may decide that the description we give depends on the circumstances.

One purpose in the development of fuzzy logic was to have some way of representing in symbolic form this phenomenon of fuzzy boundaries belonging to terms in natural language. So the theorists developed "fuzzy sets" (Gottwald 2017: §5; Zadeh 1965). A fuzzy set is defined to be identical with or correlated with a function that assigns a number 0, 1, or some real number in between 0 and 1 to each "element" that is a potential candidate to be a member of the set. An element with the number 0 is not in the set. An element with the number 1 is definitely a member of the set. And an element with the assigned number 0.3 has 0.3 membership. Or we may say that it is 30% a member of the set. By using quantities intermediate between 0 and 1, the theorist assigns partial membership. And this partial membership can itself vary from something that is only a little likely to be in the set (a value of 0.1) to something that is very likely to be in the set (say 0.9, or even 0.99). Given the idea of fuzzy sets, fuzzy logic specifies rules for reasoning about fuzzy sets and their members.

How do we evaluate the idea of fuzzy sets and fuzzy logic? In a theory of theories, we take into account the purpose of the theorists and the nature of the sphere that they are trying to represent by their theory. In this case, they are trying to represent fuzziness in meanings in natural language. None of the other systems of logic mentioned so far has this focus. So fuzzy logic ought not to be judged by whether it is innately superior or inferior to these other systems in some absolute way, independent of every context. In fact, context is all important. The context here is the context of trying to represent fuzzy meanings. Fuzzy logic surely does this representation better than the other systems, because the other systems do not do it at all. (It may be claimed that intuitionistic logic or many-valued logic tries to represent something about the fuzziness of the concept of truth. But that is different.)

Fuzzy logic may not represent fuzzy language perfectly. In fact, it does not. But it is still superior to the other systems of logic, *with respect to* the attempt to represent fuzziness. Likewise, we might observe that alethic modal logic is better at representing the logic of necessity than the other systems. When we take into account the purposes of theorists, the systems can be seen as mainly complementary.

8 Aristotle's theory of syllogisms

Let us consider a much earlier point in the development of logic, namely, Aristotle's (1938) nonmodal theory of syllogisms (Bobzien 2016: §2.4). (Aristotle also introduced the study of modal logic [Bobzien 2016: §2.5].) Aristotle's theory was an early milestone in logic. It attempts to represent pieces of reasoning in many spheres, not just mathematics. How well does it succeed?

It represented a significant piece of patterns of human reasoning in an insightful and impressive way. So it set the format for the next 2000 years of logic in the Western world. But in retrospect, we can see better what it left out. Aristotle knew that the heart of his theory did not deal directly with modal logic, temporal logic, or fuzzy boundaries in concepts. It represented only a piece – a significant piece, but still only a piece.

Moreover, even in what it did attempt to represent, it gave us a simplification or idealization. Consider a typical case of syllogistic reasoning, in the pattern called “Barbara.”

Premise 1: All mammals are animals.
Premise 2: All dogs are mammals.
Conclusion: Therefore, all dogs are animals.

If both premises are true, it follows logically that the conclusion is true.

But Aristotle knew that syllogisms with the proper *form* could be invalid if one of the terms was “equivocal,” that is, if it had more than one meaning. Consider:

Premise 1: All water is liquid.
Premise 2: All ice is water.
Conclusion: Therefore, all ice is liquid.

Both premises are true, yet the conclusion is invalid. Why? Because there is an equivocation in the word *water*. In premise 1, “water” means liquid water, as distinct from ice and water vapor. But in premise 2, “water” is used more broadly, presumably to include all three forms of water, solid, liquid, and gaseous.

To make the deductions valid, we actually have to have two special conditions, not just one. The first condition is that there is no equivocation. The second condition is that no extra context is needed to interpret the meaning of the three propositions. The meaning of each proposition has to be “independent of the world,” so to speak.

In the above example of a syllogism, this second condition is not met; the effects from context spill over and ruin the first condition as well. In the premise “all water is liquid,” the context of human experience spills over into the interpretation of the premise. The word “liquid” acts backward on “water,” and in a quick interpretation we unconsciously supply a communicative context in which people are using “water” to denote *liquid* H_2O. When we interpret the second premise, we supply a communicative context in which people are talking about the permanent composition of ice. So “water” is interpreted in terms of chemical composition, or at least in terms of the three interconvertible forms of water. Finally, with the conclusion, we interpret the proposition against the background of our knowledge of the contrast between solids and liquids, and our knowledge that the term *ice* denotes a solid. So we quickly see that the conclusion is false.

For Aristotle’s syllogisms to work correctly, there cannot be a “contamination” of meaning, a contamination spilling in from complex human knowledge and from the world that human beings know. That is the second condition. The first condition, as indicated above, is that there must be no equivocation in meaning. The meanings of the terms must remain perfectly the same in each of their occurrences.

This includes the requirement that the *boundaries* of each distinct meaning must be fixed. We cannot safely use a word like *water* if in one context “water” includes dirty water and in another occurrence it does not (Poythress 2013c: ch. 19).

9 Mathematical examples

Simple cases in arithmetic and in geometry may seem to meet these two special conditions. But that is partly because arithmetic and geometry have already been specifically tailored to have a rigidity or idealization built into them.

Let us illustrate. Let us suppose that in using the term *arithmetic* we are talking about real-world facts. When we add two more apples to a bag with two apples, we can count and find four apples. We say that means that $2 + 2 = 4$. But what we *treat as* one apple is not perfectly defined. Is it still one apple if it has a bite out of it or if it has a nick in it? And what do we mean when we speak about adding apples to a bag? When we talk about the whole process of adding apples we tacitly eliminate various special circumstances. We disallow a magician’s bag with a secret compartment. We eliminate also the special circumstance in which, after adding the extra two apples, we walk out of the room and then return, not knowing that someone else has added a fifth apple or removed one apple from the bag.

We must also consider situations in which distinct objects stick together. Suppose we add two dollops of peanut butter to two dollops that are already in the bag. We are likely to find inside the bag one big dollop!

Because of these complexities in the world, we may find it convenient to stipulate that “arithmetic,” in our treatment, is not about apples or oranges or dollops of peanut butter, but about abstract numbers: the numbers one, two, three, four, and so on. But now the abstraction has the effect of producing a special environment in which the terms *one*, *two*, etc., have a special meaning, namely, the meaning according to which they are allowed to interact *only* in this environment. We have in fact created a new semiotic subsystem. The effect is to produce exact, guaranteed results. But the effect is achieved only using a special, stipulated sphere of study, that is to say, a special *context* in which we do specific arithmetic problems. We are no longer allowing, at least as an immediate influence, the highly complex contexts that we experience in dealing with objects like apples that exist in the real world.

Similar observations hold with respect to geometry. “Geometry” could mean calculations of distances in the real world. But it often means a special sphere of study, devoted to infinitely long lines with no width, and infinitely sharp points that take up no space. If we include in our axiom system the parallel postulate, we make another step toward an artificial system, because according to Einstein’s general theory of relativity the world of physics has curvature. In the real world, parallel lines of light rays do not stay quite parallel over long distances in the presence of gravitating bodies. Thus axiomatic geometry is a distinct semiotic system, notably distinct from and at variance with real-world objects, penciled-in lines, and penciled-in points.

10 The application of a triad from tagmemic theory

Up to this point we have been using semiotics as a theory of theories in order to appreciate the structure of systems of logic. We appreciate systems of logic as semiotic subsystems by considering them in relation to a larger context, a context in the world to which they are intended to apply. And we consider these systems in relation to the theorists – the logicians who create, manipulate, and discuss them.

In considering Aristotle's syllogisms we have also noted something more specific, namely, the need to eliminate within any one syllogism possible variations in meaning and influence from contexts. These two features, variation and context, have a specific role in the theoretical apparatus of tagmemic theory. Tagmemic theory invites us to analyze any emic (Pike 1967: ch. 2, 1982: 44–45) semiotic unit in terms of three interlocking aspects: contrast, variation, and distribution (Pike 1982: chs. 6–8). These three aspects are most often employed in analyzing words, phrases, and other units of language. But they also apply to units within other semiotic systems, such as church services (Pike 1967: ch. 3), football games (Pike 1967: ch. 4), and traffic lights (Poythress 2018: §6). Let us consider how these three aspects function in a syllogism (Poythress 2021: §9.1).

"Contrast" is short-hand for the more elaborate label, "contrastive-identificational features." (Pike 1967: §3.53, 1982: ch. 6). These are the features of a specific unit that make evident the contrast of one unit from others, and also serve to *identify* and *re-identify* the unit in a positive manner in each occurrence. Contrast is operative in syllogisms in each term. For example, the term *water* must have a specific stable meaning and a specific stable form (written or oral). The meaning must remain distinct from other meanings, and must be re-identifiable in each occurrence within the syllogism.

Next, there is *variation* (Pike 1967: §3.52, 1982: ch. 7). There is a minimum kind of variation in the fact that each occurrence of *water* is distinct from all other occurrences. But notable variations in meaning occur if "water" sometimes means liquid water, and sometimes dirty water, and sometimes water in either liquid or solid or gaseous form. If a syllogism is to be purely "formal" reasoning, such variations have to be eliminated. We have to enter a specific sphere of study where we stipulate that we are temporarily evaporating or eliminating all but the most trivial variations in form, and all variations in meaning whatsoever.

Third, there is *distribution* (Pike 1967: §3.51, 1982: ch. 8). The *distribution* of the term *water* may be defined as the suite of contexts in which it is accustomed to occur. The occurrences take place within a semiotic system in which *water* functions as a unit. With a word like *water*, the most obvious contexts will be the phrases, sentences, paragraphs, and larger discourses in which it occurs. For example, *water* is a noun, so we expect it to occur in noun phrases. But *water* can also function as a verb: "I *watered* my house plants." So, like other verbs, it can be joined to the tense marked -*ed*, and can be preceded by helping (auxiliary) verbs: "I *might* water my house plants"; "I *will* water my house plants." In addition to distribution in larger discourses, we also have distribution in a larger situation: a context of persons who communicate the discourses, and a context of worlds, real or fictional or artificial, about which they communicate.

The word *water* has a complex distribution in ordinary language. Not only are there subtle variations in meaning, but these variations tie in with the distributional context of particular uses. When we say, "Water is a liquid," the distributional context contains the word *liquid*. This distributional element enables us to determine that the meaning of *water* is here the narrower meaning, which excludes ice and water vapor. This narrower meaning is within the bounds of *variation* of meaning that belong to the word *water*.

Syllogisms exist within a real-world context, within which people know what syllogisms are and how they are expected to function. This context is the context of the *distribution* of syllogisms. What kind of context is it? The people who use syllogisms have to assume, as we saw, that we can minimize or effectively eliminate *variation*. Likewise, they assume that they can minimize internal effects from *distribution*. To minimize internal effects, the theorists must construct a carefully controlled context, namely, the context of "working with syllogisms." This context is in fact a distributional context. We have this context in order to *eliminate* distribution as an extra effect that might interfere with the isolation of syllogistic propositions from the networks of meanings in the rest of the world. This effect of using distribution to eliminate distributional effects may seem strange. But it is the only way to do it.

If we were to do a comparison, we might say that the context of syllogisms is something like the context for a delicate experiment in physics. The experimental apparatus used in a specific experiment in physics may have to be physically isolated from surrounding vibrations, sounds, and light in order to detect a particular physical effect. Likewise, a syllogism has to be isolated from the effects of variation and distribution if we are to see a demonstration of maximal rigor in reasoning.

For the physics experiment, many special physical structures may be deployed in the environment of the core experimental apparatus. We may, for example, see sound-proofing in the walls to dampen the effects of sounds coming from outside the room. These structures offer a distributional context that serves to minimize the distributional physical effects on the core apparatus. Likewise, with syllogisms, we put in place special conditions in the discourse environment and the larger communicative environment. Only then do we proceed to examine the core apparatus, namely, the syllogisms themselves.

With the help of physical isolation, the experimenter may hope to gain insight into some "deeper," more ultimate behavior in the physical world. Does the pattern of a syllogism give us deep insight into the nature of logic and reason? Maybe. But also maybe not. It depends on what we consider to be "deep," as well as a possible influence from a larger worldview that tells us how to get at what is deep.

In fact, there are both similarities and differences between physics and logic. Physics focuses on researching an aspect of the world that seems to exist independent of humanity. The atoms and the stars would exist even if we were not here to observe them. It could be argued that logic also exists independently of the world. We reason not simply as a game, but to draw conclusions about the world. If so, the world must have a certain rationality that matches our reasoning about it.

And yet the development of theories of logic depends a lot on observations about language. Aristotle's categorical syllogisms all have premises that take one of four linguistic forms: either "every A is B" or "some A is B" or "no A is B" or "some A is not B" (Parsons 2017: §2). The patterns of the syllogisms imitate certain patterns of reasoning that we find in human language. So logic aims at representing reasoning in language as much as it represents reasoning about the world. In individual human experience, practical reasoning in language always precedes the more specialized discussion of syllogisms or other special logical patterns. A larger environment of language shapes the special circumstances of "isolation" that are assumed when we examine syllogisms.

The central question, then, is whether formal logic has succeeded in representing only a *fragment* of reasoning, rather than its whole scope. The answer suggested by our analysis with tagmemic theory is that it represents a fragment. In general, emic units of meaning have contrast, variation, and distribution in a complex, interlocking whole. The syllogism and other forms of formal logic can be constructed only by producing special semiotic subsystems that are tailored specifically to mask or suppress the influence of variation and distribution.

Human communication, and human communication using *reasoning*, is *richer* than what is captured in syllogistic forms. This principle holds not only with respect to human communication in general, but with respect to scientific theories. The theories have to interact with the world in complex ways, and the scientists who form the theories have to exercise complex personal judgments. Their judgments are influenced by background knowledge (Polanyi 1958).

Analogous results come up if we consider the general principle of the contribution of persons to theory formation. The persons are more complex than the resulting theories.

11 The assessment of contexts

In short, systems of formal logic work well because we, as full persons, are able insightfully to shape special contexts in which the systems work with their full force. We are at least subliminally aware of the larger framework in which a system of logic sits. Using this framework, we can see, in a particular case, that the framework has managed to eliminate the influence of variation and distribution. Along with this elimination comes the intensification of the all-important aspect of contrast. Each term in the formal system is what it is, in lonely contrast to everything else. Since we are aware of this framework, we can assure ourselves, by repeated testing as well as by more intuitive general observations, that the system "works." It gives us a supreme rigor, and at the same time it to some degree matches reasoning in ordinary language to which it corresponds.

The situation with special formal logics suggests by analogy what happens in reasoning in ordinary language. We are guided by a larger context. The larger context gives us clues as to whether one particular case of reasoning is sound. But we are never going to be able to formalize all of this context. Formalization is a special case, not the standard for everything else.

12 A mathematical model for variation and distribution

For those who love to have formal models, we may offer this consolation: we can have a formal, mathematical model to represent more rigorously our observations earlier in this article. The model can include a representation of contrast, variation, and distribution. It can include the idea of theories with a larger context, and the idea of theories of theories. It can represent nearly everything that we have discussed above. But any such model is not ultimate. Rather, it is selective. We, as theorists who are also persons, are still the ones who have to do the selection.

A model of this kind has already been largely worked out in previous work on semiotics (Poythress 2013a, 2013b, 2015). In a manner similar to a lattice model of quantum mechanics (Ismael 2015: §4; Wilce 2017a: §1, 2017b), the model reduces all the propositions in a first-order theory and/or a higher-order theory to yes-no questions. Each question has associated with it a *probability* that it will turn out to be true. Questions that are known to have a "yes" answer have a probability of 1. Questions that are known to have a "no" answer have a probability of 0. *Contrasts* are cases where two questions cannot both have a "yes" answer. *Variations* are cases of conditionality, where a "yes" answer to one question (let us say a particular occurrence in communication that has sufficient features associated with the word *water*) implies a "yes" answer to a "supervening" question, namely, whether the particular occurrence is an instance of a general category (the word *water*). Distribution concerns cases where two questions have probabilities that are roughly independent of each other. But there may be a partial dependence in some cases.

With the addition of some other features, the model as a whole is a mathematical lattice with an associated "probability measure." The lattice structure arises because any two yes–no questions *A* and *B* can be combined using "and" to produce another question, "*A* and *B*." The combined question has a yes answer if and only if *A* and *B* both have a yes answer. The resulting combined question is represented in the lattice as the "meet" [$\wedge$] of the lattice points for *A* and *B*. Similarly, the combination "*A* or *B*" (nonexclusive "or") is represented by the "join" [$\vee$] of the lattice points for *A* and *B*.

Within a lattice that is constructed to represent a second-order theory, distinct first-order theories can be represented by sublattices. Each first-order theory has its distinct suite of questions, represented by points in its sublattice. Each sublattice for a particular theory is selected out by a suitable cluster of "yes" questions that are *outside* the sublattice, but which together specify the conditions under which the sublattice is functioning. (The "projection" associated with the cluster of "yes" questions reduces the total lattice to the relevant sublattice [Wilce 2017a: §§1.1–1.2].) By similar means, we can represent third-order and fourth-order theories within a large enough lattice. The lattice as a whole can thus encompass multiple first-order theories. Each theory has its own suite of questions. The theories do not need to be seen as competitive, because we are free to ask questions belonging to any one of the first-order theories, in their application to any particular domain where theorists apply them. For example, we can have a suite of questions about the truth or falsehood of syllogistic propositions, to represent Aristotelian logic. We can likewise have a suite of questions concerning statements of necessity about the world, and correlated questions about logicians who are using formal notations in modal logic in order to represent these necessities.

Together, this suite of questions is a representation of modal logic. The same principle holds if the lattice includes multiple theories of higher orders.

In these cases, we should note that first-order theories are supposed to be treated as *applied* theories. They are not simply abstract systems of signs with abstract rules for manipulating signs. They are associated with formal models or arenas within the real world where they are supposed to apply. There will be a distinct sublattice for each distinct arena of application.

The first-order theories apply well when the objects referred to by the theory have emic units that exhibit sharp contrasts and very little variation or distributional influence from questions *outside* the objects, questions that would specify an environment. On the other hand, when there *does* exist influence, it means that the objects studied are not suitably "isolated." We have here an analog to physical interference in a subsystem through light or noise, or an analog to the effect of quantum entanglement. The *lack* of interference, either for a physical subsystem or for the objects of a first-order theory represented by a sublattice, represents a kind of ideal case for the application of the theory.

Such a representation of multiple subsystems of logic does not capture everything. But it is enough to confirm that the kind of reasoning we are doing in this article is rational. And, by using a richer form of model, it confirms the impression that Aristotle's system of syllogisms and the other special logics that we have mentioned do not offer us an ultimate standard for all human reasoning. Rather they, as well as the lattice model, operate *within* a pattern of human reasoning that is richer than any one of them or even all of them together.

References

Aristotle. 1938. *Categories; On interpretation; Prior analytics*. Cambridge: Harvard University Press.

Bobzien, Susanne. 2016. Ancient logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Winter 2016 edn. https://plato.stanford.edu/archives/win2016/entries/logic-ancient (accessed 19 March 2020).

Bridges, Douglas & Erik Palmgren. 2015. Constructive mathematics. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Summer 2018 edn. https://plato.stanford.edu/archives/sum2018/entries/mathematics-constructive (accessed 28 March 2020).

Cintula, Petr, Christian G. Fermüller & Carles Noguera. 2017. Fuzzy logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Fall 2017 edn. https://plato.stanford.edu/archives/fall2017/entries/logic-fuzzy (accessed 18 March 2020).

Copi, Irving M. 1979. *Symbolic logic*, 5th edn. New York: Macmillan.

Garson, James. 2018. Modal logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Fall 2018 edn. https://plato.stanford.edu/archives/fall2018/entries/logic-modal (accessed 18 March 2020).

Gödel, Kurt. 1931. Über formal unentscheidbare Sätze der *Principia mathematica* und verwandter Systeme, I [On formally undecidable propositions of *Principia mathematica* and related systems]. *Monatshefte für Mathematik und Physik* 38. 173–198.
Gödel, Kurt. 2000. On formally undecidable propositions of *Principia mathematica* and related systems, Martin Hirzel (trans.). https://web.archive.org/web/20040916041216/http://www.research.ibm.com/people/h/hirzel/papers/canon00-goedel.pdf (accessed 21 March 2020).
Gottwald, Siegfried. 2017. Many-valued logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Winter 2017 edn. https://plato.stanford.edu/archives/win2017/entries/logic-manyvalued (accessed 18 March 2020).
Halbach, Volker & Graham E. Leigh. 2020. Axiomatic theories of truth. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Spring 2020 edn. https://plato.stanford.edu/archives/spr2020/entries/truth-axiomatic (accessed 28 March 2020).
Iemhoff, Rosalie. 2019. Intuitionism in the philosophy of mathematics. In Edward N. Zalta (ed.), *The Stanford encyclopedia of Philosophy*, Winter 2019 edn. https://plato.stanford.edu/archives/win2019/entries/intuitionism (accessed 18 March 2020).
Ismael, Jenann. 2015. Quantum mechanics. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Spring 2015 edn. https://plato.stanford.edu/archives/spr2015/entries/qm (accessed 18 March 2020).
McNamara, Paul. 2019. Deontic logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Summer 2019 edn. https://plato.stanford.edu/archives/sum2019/entries/logic-deontic (accessed 18 March 2020).
Moschovakis, Joan. 2018. Intuitionistic logic. In Edward N. Zalta (ed.), *The Stanford Encyclopedia of philosophy*, Winter 2018 edn. https://plato.stanford.edu/archives/win2018/entries/logic-intuitionistic (accessed 18 March 2020).
Parsons, Terence. 2017. The traditional square of opposition. In Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Summer 2017 edn. https://plato.stanford.edu/archives/sum2017/entries/square (accessed 21 March 2020).
Peirce, Charles S. 2011 [1897–1910]. Logic as semiotic: The theory of signs. In Justus Buchler (ed.), *Philosophical writings of Peirce*, 98–119. New York: Dover.
Pike, Kenneth L. 1959. Language as particle, wave, and field. *The Texas Quarterly* 2(2). 37–54.
Pike, Kenneth L. 1967. *Language in relation to a unified theory of the structure of human behavior*, 2nd edn. The Hague & Paris: Mouton.
Pike, Kenneth L. 1976. Toward the development of tagmemic postulates. In Ruth M. Brend & Kenneth L. Pike (eds.), *Tagmemics: Volume 2: Theoretical discussion*, 91–127. The Hague & Paris: Mouton.
Pike, Kenneth L. 1982. *Linguistic concepts: An introduction to tagmemics*. Lincoln, NB & London: University of Nebraska Press.
Polanyi, Michael. 1958. *Personal knowledge: Towards a post-critical philosophy*. Chicago: University of Chicago Press.
Poythress, Vern S. 2013a. An information-based semiotic analysis of theories concerning theories. *Semiotica* 193(1/4). 83–99.
Poythress, Vern S. 2013b. Information-theoretic confirmation of semiotic structures. *Semiotica* 193(1/4). 67–82.
Poythress, Vern S. 2013c. *Logic: A god-centered approach to the foundation of Western thought*. Wheaton, IL: Crossway.

Poythress, Vern S. 2015. Semiotic analysis of the observer in relativity, quantum mechanics, and a possible theory of everything. *Semiotica* 205(1/4). 149–167.

Poythress, Vern S. 2018. A simple traffic-light semiotic model for tagmemic theory. *Semiotica* 225(1/4). 253–267.

Poythress, Vern S. 2021. Semiotic analysis of symbolic logic using tagmemic theory: With implications for analytic philosophy. *Semiotica* 243(1/4). 171–186.

Raatikainen, Panu. 2018. Gödel's incompleteness theorems. In Edward N. Zalta (ed.), *The Stanford encyclopedia of Philosophy*, Fall 2018 edn. https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/ (accessed 21 March 2020).

Rosser, John Barkley. 1939. An informal exposition of proofs of Gödel's theorem and Church's theorem. *Journal of Symbolic Logic* 4. 53–60.

Shapiro, Stewart & Teresa Kouri Kissel. 2018. Classical logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Spring 2018 edn. https://plato.stanford.edu/archives/spr2018/entries/logic-classical (accessed 18 March 2020).

Wilce, Alexander. 2017a. Quantum logic and probability theory. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Spring 2017 edn. https://plato.stanford.edu/archives/spr2017/entries/qt-quantlog (accessed 18 March 2020).

Wilce, Alexander. 2017b. Supplement to quantum logic and probability theory: The basic theory of ordering relations. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Spring 2017 edn. https://plato.stanford.edu/entries/qt-quantlog/supplement.html (accessed 18 March 2020).

Zadeh, Lofti A. 1965. Fuzzy sets. *Information and Control* 8. 338–353.