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Semiotic analysis of symbolic logic using tagmemic theory: with implications for analytic philosophy

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Abstract: This article uses tagmemic theory as a semiotic framework to analyze symbolic logic. It attends particularly to the issue of context for meaning and the role of personal observer/participants. It focuses on formal languages, which employ no ordinary words and from one point of view have "no meaning." Attention to the context and the theorists who deploy these languages shows that formal languages have meanings at a higher level, colored by the purposes of the analysts. In fact, there is an indefinitely ascending hierarchy of theories of theories, each of which analyzes and evaluates the theories at a lower level. By analogy with Kurt Gödel's incompleteness theory, no level of the hierarchy can capture within formalism everything in a sufficiently complex system. The personal analysts always have to make judgments about how a formalized system is analogous to the world outside the system. Arguments in analytic philosophy can be useful in clarification, but neither clarification of terms nor clarification of the structure of arguments can eliminate the need for personal judgment.

Keywords: analytic philosophy; hierarchy of theories; symbolic logic; tagmemic theory

A simple definition of "symbolic logic" might say that it is logic using special symbols. For example, instead of the word *and* it uses the now-standard symbol for logical "and," namely "∧" (U2227, "wedge," "conjunction"). Instead of the word *or* it uses the standard logical symbol "∨" (U2228, "vee," "disjunction"). Each symbol has a written representation, and at the same time it has a meaning within the field of logic. Thus, each special symbol in symbolic logic is a "sign" in the sense of semiotics. We propose to consider symbolic logic as an instance of a sign system, and to study it using the tools of semiotics.

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1 Symbolic logic as a field of study

We should first consider what is the scope of the field of symbolic logic. It is complicated. Symbolic logic has ancient roots. But it came to flower mostly in the twentieth century (Bochenski 1961; Gabbay and Woods 2004; Haaparanta 2009). The study of the logic involved in mathematical proofs led to formalization and the production of a suite of special symbols. With the aid of these symbols, many standard proofs in mathematics and in logic proper can be written out just with symbols, with no aid from words of ordinary language. (A classic milestone was Whitehead and Russell 1910–1913.)

Symbolic logic is represented not by one single system, but by a cluster of related systems. What is common to these systems is an interest in formalized representation of logical operations and deductions, using special symbols. There is a spectrum with respect to the *degree* of formalization. At one end are systems with no words of ordinary language left in them. At the other end, formal logical symbols can be used in a piecemeal way within paragraphs of reasoning that include ordinary language.

As an example, we may use the first three lines of Kurt Gödel's (1995 [1970]) ontological proof of the existence of God:

Axiom 1. $P(\varphi){\cdot}P(\psi) \supset P(\varphi{\cdot}\psi)$. [footnote: And for any number of summands.]

Axiom 2. $P(\varphi) \vee P({\sim}\varphi)$. [footnote: Exclusive or]

Definition 1. $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

This is mostly just special symbols. But there are still a few ordinary words left, which explain what is some of the context for the symbols.

Special symbols can be used in treating more than one kind of content. The most common content is either mathematics or general formulations about deductive reasoning. But symbolic logic can also be deployed by analytic philosophy. Analytic philosophy can study many fields of scholarship, with the intent of clarifying and/or purifying and refining the reasoning. So we find analytic philosophy offering deductive reasoning in epistemology or ethics or philosophy of science or philosophy of religion. The individual propositions are typically formulated primarily in ordinary language, but arguments connecting the propositions may be formulated using some apparatus from symbolic logic. In this article, our final implications will include attention to this kind of analytic philosophy.

2 Tagmemic theory as a tool for semiotic analysis

We use *tagmemic theory* to conduct our semiotic analysis. Tagmemic theory (Pike 1967, 1982) is commonly known as a theoretic framework for studying natural languages. But it is in fact crafted as a general theory for meaning in human behavior of all kinds. It is thus a semiotic theory, not merely a narrowly linguistic-only theory (Poythress 2013a).

Tagmemic theory is an attractive tool for at least two reasons: (1) It pays attention to and emphasizes the larger human context in considering any system of signs; (2) It pays attention to the persons involved. The personal participants are not excised from the system (Pike 1982: 3–4). These points (1) and (2) are complementary to each other. The context includes human beings. And the human beings who participate in semiotic interaction are tacitly aware of a larger context, outside the focus of their immediate acts of communication (Polanyi 1958, 1964, 1967, 1975).

We can put it another way. Sign systems are never merely "bare" notations, but systems used by *people*. Unlike some more formalized approaches to linguistics, tagmemic theory explicitly includes the human participants and asks about the meaning of their participation. What meanings do the participants themselves grasp? And what meanings may be communicated to others outside the immediate circle of action? The principle of including the observer applies *both* to ordinary users of signs and to theorists, that is, people who formulate theories about the users of signs. Moreover, tagmemic theory affirms the possibility of multiple points of view on the part of linguists and semioticians (Pike 1972 [1959], 1982). The theorist is considered in relation to the theory; any theory, whether semiotic or scientific, does not stand by itself, but is a theory in relation to the personal purposes of those who craft it and those who use it. And multiple purposes are possible.

3 The meaning of formalization

Using this starting principle of paying attention to persons, we can already see a simple application of tagmemic theory to symbolic logic. Symbolic logic exists because people created it. People – human beings – are irreducibly involved, and when they do something meaningful, the meaning is connected to personal intentions. They *generate* new meanings. Historically, each special symbol, like the wedge symbol $\wedge$ for logical "and," had to be introduced by a person or persons who specified its meaning and its graphical and oral form. In writing this article, I myself have had to re-specify the wedge symbol and explain it to any readers who are not already familiar with it.

Moreover, the specification of meaning depends on *context*. The contexts may include both explicit and implicit contexts. In my partial explanation of the wedge symbol $\wedge$, I said that it symbolized “logical ‘and.’” But that explanation relies implicitly on the literature and tradition of formal logic, which explains in greater detail just what “logical ‘and’” is and just how it functions. This tradition of formal logic has an even larger context, according to which people have purposes in mind when they develop formal logic. They want to use the logic in some way. And that way relates eventually to meanings in natural language and meanings in the world.

We also need to acknowledge that there is a kind of syntactic context for the use of the wedge symbol. The wedge symbol is appropriately used when it links two propositions. We may illustrate by an example:

snow is white $\wedge$ 1 + 1 = 2.

“Snow is white” is a proposition. And “1 + 1 = 2” is a proposition. The joint proposition, with the key linking symbol $\wedge$, means that snow is white *and* 1 + 1 = 2. In a sense, the wedge symbol $\wedge$ does not yet have a function if it just sits by itself. It has meaning only in a combination of the form “$A \wedge B$”, where A and B are propositions. It needs *context*, the context of A and B, to make sense.

All of this is elementary, and is mostly taken for granted by those who use symbolic logic. But it is worth noting it, when we are ready explicitly to reflect about the significance of context and the significance of persons who intend to do things with signs.

4 Formal languages – without meaning?

Within the field of symbolic logic, at the extreme end of the spectrum of degrees of formalization are systems that are called “formal languages.” These systems are *completely* formalized. Formal languages have several defining features. To begin with, they are semiotic systems that contain no instances of words taken from natural languages. They contain only artificially constructed symbols like the wedge symbol $\wedge$. They may also have a collection of special symbols like A and B that can function in syntactical arrangements like $A \wedge B$. The expression “$A \wedge B$” is a *sequence* of signs.

But formal languages are special not only because they do not have ordinary words. They also do not have ordinary *meanings*. They are deliberately constructed to “have no meaning” in a certain sense. They consist of signs that are treated as technically meaningless, but that fit together according to certain fixed formal rules.

For the benefit of readers not already familiar with this concept, let us spell out how it works in a typical case of a formal language. Typically, the rules of a particular formal language are of four types. (1) The first type of rule specifies what are the atomic signs, that is, the signs that are treated as wholes that cannot be further divided. Consider the above example of "$A \wedge B$." A, $\wedge$, and B are atomic signs. "$A \wedge B$" is *not* an atomic sign, but a sequence of three atomic signs. (2) The second type of rule specifies what sequences of atomic signs are "terms" (which are comparable to nouns in natural languages). (3) The third type of rule specifies what sequences of terms and other signs count as "propositions." (These propositions are analogous to clauses and sentences in natural languages.) (4) The fourth type of rule specifies what sequences of propositions are "proofs." (These "proofs" are analogous to sequences of sentences in natural language, where each sentence functions as one stage in a deductive argument.) The rules of the fourth type have two subcategorizations: (a) rules specifying what are the "axioms"; (b) rules specifying what steps are allowed in a proof sequence in which other propositions are shown to be deducible from the axioms. The axioms are analogous to sentences in natural languages that are assumed to be true at the beginning of an argument.

It should be obvious that this kind of formal language imitates various aspects of what takes place in proofs within mathematics. The atomic signs in a formal language are analogous to the smallest mathematical signs (Numerals like "1" or "2" are atomic signs in mathematics; so is the plus symbol "+".) The terms in a formal language are analogous to sequences of signs used in mathematics to denote mathematical objects. For example, the sequence of numerals 1, 0, 0, 1 is used to designate the number 1001; "1 + 3" is one designation for the number 4. The propositions in a formal language are analogous to claims about mathematical truth. For example, "1 + 1 = 2" is a proposition (which happens to be true). "1 + 3 = 5" is a proposition (which happens to be false).

The formal proofs are analogous to what we call proofs in mathematics. More loosely, we can also observe analogies between these formal proofs and arguments of various kinds within natural languages – arguments about scientific theories or politics or ethics or metaphysics or anything of human concern.

But there is a special feature that belongs to a formal language. It is not supposed to be "about" anything at all. It is a "stripped down" language. It is nothing but a system of rules for manipulating signs and sequences of signs. It is not "about" logic or arithmetic or politics or anything true or false about the real world.

5 Appropriateness of semiotic analysis of formal languages

But if a formal language does not mean anything at all, can it legitimately be an object of study for semiotics? We need to consider this question carefully. Semiotics studies "systems of signs," but the signs are always signs *of* something or other. They "signify." Typically, semiotics deals with systems with a dual articulation – form and meaning. Early in the historical development, Ferdinand de Saussure (1916, 1998) explained it. A sign is a unit that possesses both form and meaning. In his terminology, a sign combines (1) a signifier, which is the *form*; and (2) what is signified, which is the *meaning*. For example, the sign *dog* in English includes form and meaning. The form consists in a sequence of sounds or letters: *d-o-g*. The *meaning* of the word *dog* is "domestic canine mammal."

A formal language has only half of this combination. Within the formalism there is something akin to a signifier, namely the atomic signs like the wedge symbol $\wedge$. Sequences of signs can also function as compound instances that function like signifiers. But there are no signifieds. This lack is deliberate. We could sum it up by saying that there are forms but no meanings. If so, is a formal language a suitable object for semiotic analysis?

The answer is yes. But to see why, we have to engage in a more fine-tuned analysis. The materials in a formal language have no first-order meaning; they do not function as units that combine the aspects of the signifier and the signified in the ordinary way. But these formal languages are created and used by human beings, namely those who theorize about formal languages. These human beings create meaning, find meaning, and communicate meaning in the process of talking about and employing formal languages. Even if formal languages were merely light-hearted recreational games that people would play to practice symbol manipulation, they would still have human meaning as *games*. The meaning here is a kind of second-order meaning, the meaning imparted by the people who are thinking about formal languages. Without such a participation, formal languages would not be a part of the broad human cultural project at all. From a human point of view, they would then be fully meaningless, because we would never be talking about them.

So why have theorists created formal languages in the first place? There are several interlocking reasons. Considering these reasons leads us into a position where we can properly deploy a semiotic analysis. We may consider two main reasons.

First, like many instances in not-fully formalized logic and mathematics, theorists have an interest in generalization. They see common patterns in many instances of reasoning. So they try to capture the commonality with a general notation. We may illustrate by using a parallel example in mathematics. In mathematics, we may notice that the operation of addition is "commutative": $1 + 3 = 3 + 1$; $2 + 3 = 3 + 2$; $4 + 6 = 6 + 4$; and so on. We generalize by substituting two general symbols a and b for particular numerals. $a + b = b + a$. This expression of the commutative property is a generalization.

Now let us return to the topic of logic. Aristotle thought that he saw general patterns for making valid deductions. His theory of the syllogism was meant to capture and codify these general patterns (Aristotle 1938 [originally fourth century BC]). These syllogistic patterns apply to cases of reasoning about many distinct topics.

So we may suggest that formal languages represent a case of generalization. The signs *seem* to have no meaning, because they are so general. For example, in the expression $A \wedge B$, A and B are not specific propositions, such as "snow is white" or "$1 + 1 = 2$." They represent propositions of any kind, in a very general way. They have "no meaning" to begin with. But they are ready to be filled with meaning, once we illustrate the general pattern by applying it to a particular case.

There is then a kind of interplay between "no meaning" and "meaning." Roughly speaking, A and B have "no meaning," or, more precisely, very little specific meaning, only "potential" meaning, *until* we produce a specific meaning by substituting into the expression specific propositions like "snow is white."

This interplay between no meaning and meaning involves both observers and contexts. Tagmemic theory, by its attention to observers and contexts, encourages us to find meaning. The meaning does not belong to a formal symbol like A or the wedge symbol $\wedge$ *in isolation*. We find meaning when we observe how in practice observers or theorists deploy the symbol A. They illustrate using examples like "snow is white." Or they explain to us that eventually the formal system will be seen as relevant to a context in which we relate it to propositions or other features belonging to the world of mathematics, the world of nature, or the world of human rational arguments. Meaning is there. We can even say that there is meaning in the symbol A, if we like. But the meaning has been moved primarily into the context and into the explanations given by the theorist. The theorist tells us, who are semiotic participants along with him, why we ought to be interested in this seemingly meaningless "game" of symbol manipulation. We ought to be interested because in fact the system of the formal language is not constructed arbitrarily. It is not a mere fancy of the mind. It is constructed in order to be used. It will be seen as relevant to another context outside of the formalism itself.

The second reason for deploying formal languages is in order to examine with great rigor what can and cannot be deduced from a certain set of starting axioms. To achieve such rigor, the rules for deduction must be made purely syntactical.

They must not depend on the meanings of the propositions. Once the system is reduced to a purely syntactic system, a completely rigorous, mathematical kind of argument might be deployed in order to show which propositions are deducible within the system. And in fact such a thing has been done for the standard formal logic of propositions, and for the standard formal logic of "first order quantification" (Copi 1979). With a suitable choice of axioms, one may show in a mathematically rigorous way that all tautologies can be deduced within the system, but nothing else. In other words, the system allows the deduction of all the propositions that intuitively we think ought to be deducible, and nothing else besides. It is an impressive result. And it depends, let it be noted, on a technique that at least temporarily sets aside the main questions about meaning. The system is *temporarily* treated as if it were a pure notational system, with no relation or relevance to the real world.

But such a treatment obviously depends on human participants who in fact know what they are doing. These human participants know that, in the long run, the purely formal language that they study is not going to *remain* purely formal. Later on, the symbols will be invested with meaning of some kind. They will apply to some portion of mathematics, or perhaps even to deductive arguments in science or in metaphysics.

Semiotic analysis, then, is appropriately applied to these formal languages, even though they are "purely" formal. A semiotician appropriately notes the special, purely syntactical nature of the formal language. But, more importantly, he may also study the purposes of the theorists who create and analysis this formal language. The meanings are to be found primarily in the theorists themselves, as observers of the semiotic system that they have created. And meanings are to be found in the contexts to which the formal language or languages may later be applied. Finally, the theorists who work with formal languages are themselves participants in robust communication with their fellow theorists. So we may examine semiotically the communication that takes place within a research group dedicated to formal languages.

6 A hierarchy of theories

It is of considerable interest to note here the presence of a growing hierarchy of theories (compare Poythress 2013a). On the bottom rung of the hierarchy is a formal language or a collection of formal languages. The next rung of the hierarchy is occupied by participants who create the formal languages and produce theories about them. As we observed, one of their main goals may be to demonstrate rigorously what propositions can be proved within a particular formal language.

The third rung of the hierarchy belongs to people who analyze the participants who occupy the second rung. What we are doing in our semiotic analysis of the theorists of formal languages belongs to this third rung. This analysis of the second rung by the third rung of course extends to include also the formal languages (on the first rung) about which the participants theorize. That is to say, the people on the third rung are analyzing the people on the second rung and in addition everything on the first rung to which the people on the second rung pay attention. This analysis distinguishes carefully between the formal language on the one hand and, on the other hand, the meanings that the participants give to their formal language in the course of observing analogies between the formalism and what takes place in the kind of proofs that engage most mathematicians.

Especially in the twentieth century, the third rung began to be occupied by some mathematicians. These mathematicians began to focus on the foundations of mathematics. In the face of certain paradoxes that turned up in naive set theory, mathematicians became uneasy about whether the standard foundations of mathematics could be shown to be consistent. David Hilbert, especially, rose as a key figure in "metamathematics," the study of axioms and proofs (Mancosu 1998; Reid 1996). He proposed the goal of using a study of formal languages to establish that the starting assumptions of mathematics could be rigorously guaranteed never to lead to contradiction. Hilbert here was occupying the third rung, because he was studying how the work of mathematicians on mathematics as a whole had relations to the study of formal systems. The formal systems are the first rung; the study of formal systems (treated as themselves meaningless) is the second rung. And Hilbert's study of the study of formal systems is the third rung.

7 The failure of the program of securing mathematical foundations

Hilbert and others with him had high hopes that they could succeed. It was something of a shock when Kurt Gödel obtained a contrary result in the theory of proofs. Gödel (1931, 1992) showed that the quest for guaranteeing mathematical truth could not be fulfilled. Gödel's original paper contained considerable technical complexities (Nagel and Newman 2008). But at the heart of it lay a clear distinction between the formal system that Gödel analyzed and the meanings of that system, when treated not as a purely formal system but as terms and propositions about arithmetic. Gödel represented the formal system in ordinary arithmetic, using an arithmetic coding system for the individual signs and sequences of signs. So there was a system of coded correlations between ordinary numbers on the one hand and propositions and proofs in the formal system on the other hand. Roughly speaking, Gödel was then able, using the signs belonging to the formal language, to produce a proposition about arithmetic whose meaning *outside* the formalism was “I am unprovable” (Poythress 2013b: 592–598).

The proof depended in an essential way on the interaction of two distinct rungs within the hierarchy of theories. The formal system itself, viewed as purely syntactic, belonged to the first, lowest rung. Gödel’s arithmetic coding system, viewed as a meaningful representation of the syntax, was a kind of higher-level analysis of the system. That is, it belonged to the second rung. Gödel in analyzing the coding system stood on the third rung. Gödel’s result suggested a further generalization. We should expect that any formal system that is rich enough to include syntactic resources for describing ordinary arithmetic would display the same basic difficulty. The system would be either inconsistent (generating contradictions), or incomplete. If it were inconsistent, it would generate too many theorems (in fact, a single contradiction leads to making all propositions deducible). Or else, if it were consistent, it would generate too few theorems. It would be incomplete because one could find a proposition that was true but not provable. Gödel’s own analysis showed the way in which one could actually produce such a proposition that was unprovable within the system, but could be seen informally to be true when one considered its meaning.

This situation shows the vital importance of meaning, and the importance of a hierarchy of theories. Some formal languages may themselves contain notations for denoting a hierarchy of classes or of terms (Coquand 2018; Kamareddine et al. 2004). But it is always possible for a theorist, who creates the formal language, to stand outside of it and to see its limitations. The hierarchy of theories cannot be closed off from the top, with a final formalization of everything below it, in such a way as to capture all the truths of arithmetic.

Instead, as tagmemic theory reminds us, the observer cannot be eliminated. Formalization at one rung of the ladder of the hierarchy of theories always depends on the work of a person who is *doing* the formalization. Meaning disappears out of the formalized signs only through the meaning-filled work of the one who is doing the formalizing, or who afterwards analyzes and gives significance to the formalized system.

8 Implications for formalized arguments in analytic philosophy

We can now explore possible implications for the process of formalization in the techniques of analytic philosophy. One common technique is to try to “purify” or "refine" the logic involved in a substantive argument by moving at least part way to formalized signs. This process can be useful analyzing the structure of the argument and examining whether there are flaws.

But there are limitations. The value of formalization depends on retaining a relation between the more formalized version of the argument and the less formal modes of argumentation that occur in many spheres of life. We as analysts can always stand back and ask ourselves whether the formalized version of the argument adequately represents what is going on in ordinary human language and ordinary human thought within a larger context. The context, in the end, includes the world. And the world as a whole is not formalizable.

So a formalized argument can never actually stand by itself. It needs an analyst to articulate its meaning in relation to the world as a whole. And we are always capable of questioning, at a metalevel of argumentation, whether the semi-formalized system actually matches the world. That judgment is a judgment that has to be carried out by persons. The person who creates a formalized argument is operating at least on the second rung of analysis, because he is standing over the process of formalization. The person who then questions whether the newly created formal system is actually analogous is on the third rung, because he is analyzing the quality of the work of the person on the second rung.

In fact, we find a situation here that is at least vaguely analogous to what happens with Gödel's proof of the incompleteness of certain formal systems in mathematics. Gödel uses his third-rung position to show the incompleteness of a second-rung interpretation of a formal system. This incompleteness applies to any formal system rich enough to include ordinary arithmetic.

But within the hierarchy of theories in analytic philosophy, we can represent the natural numbers. We have spoken of the first, the second, and the third rung. This is a numeral sequence. Clearly it can be extended, because we can always imagine an analyst on the next rung up, who undertakes to analyze the quality of the work belonging to the level below him. The rungs extend up potentially to infinity. The whole number system is represented by the hierarchy of theories. And then, if we attempt to grasp the whole sequence of theories, we have the capacity to analyze the arithmetic belonging to these rungs. This system is rich enough to include ordinary arithmetic, and so Gödel's result suggests that a formalization of arguments in analytic philosophy can never be complete. It can never capture everything that a human analyst is capable of deducing. Moreover, we are always capable of asking one decisive question about the implications of a semiformalized argument. Does the argument in its formalized structure actually correspond appropriately to ordinary language arguments about the real world?

Any extra benefits of formalization depend on there being a significant distinction between the formalization and the starting informal case. And because there is a distinction, the formalized and unformalized structures may not actually match in the appropriate way.

9 Implications for formalized *terms* or *concepts* in analytic philosophy

A second technique in analytic philosophy is to attempt to refine our understanding of key *terms* or key *concepts*. This task is complementary to the task discussed above, of refining *arguments*. Like the preceding task, this task can be useful. We may find that a certain key term in the world of informal argument is ambiguous or equivocal. Perhaps it oscillates between two related but distinct meanings. So arguments that slide unwittingly between the two meanings are invalid. An analytic philosopher may point out such an ambiguity, and tell us to stick to only one meaning in the course of argument. Or a key term, without being equivocal, can still be vague. It may, in fact, be too vague to communicate much by way of truth. So a philosopher undertakes to substitute a more precise term or a precise definition of the previously vague term.

All this, as we say, can be useful. But tagmemic theory has resources that highlight the limitations involved in the clarification or refinement of concepts.

9.1 Aspects of emic units

Let us begin with tagmemic analysis of units. Tagmemic theory postulates that emic semiotic units involve inextricably three aspects – contrastive-identificational features, variation, and distribution (Pike 1982: 42–65). Contrastive-identificational features positively identify a unit as distinctive, and also separate it (by contrasts) from other units that do not have the same features. This contrastive-identificational aspect is usually heightened and made more exact when someone attempts to clarify a concept. The second aspect in semiotic units is variation. This includes possible variations in *meaning*. A purified concept typically tries to suppress variation. Similar observations about suppression hold for the influence of "distribution," that is, the spectrum of expected contexts in which a particular unit appears in particular ways. A purified concept should typically be exactly the *same* in all of its occurrences.

But note that a purified concept inevitably *differs* from its starting point in a vague concept or widely used word or phrase in ordinary language. It differs precisely because contrast, variation, and distribution are "adjusted" in the course of purification or re-definition. So the purified concept can never perfectly match the starting point. The question is always appropriate as to whether the purification actually offers insight about the world, or whether it just changes the subject by not matching the vague concept with which we started.

9.2 A hierarchy of analysis

In addition, the procedure of clarifying concepts has limitations parallel to the limitations that we observed in the previous section with respect to arguments. When we introduce a new, precisely defined term, or when we try to give to a vague term a more precise meaning, we are depending on observations about the relation of the new terms to things in the world, or things earlier conceptualized in human discussion. The comparison of these two spheres takes place by means of a hierarchy, similar to our hierarchy of theories. We stand back from each of the spheres of conceptualization, namely, the newer clarified use and an older conceptualization, in order to compare them. This process of standing back involves ascending to the next higher rung on the hierarchy. It is we, as people standing on the third rung, who evaluate the work done by people and the terms on the second rung. And then our own attempt on the third rung to evaluate the second rung can in turn be evaluated by people who compare our evaluation with the things on the second or lower rungs, things previously evaluated on the third rung. These new evaluators stand on the fourth rung in order to evaluate our work on the third rung.

As in the case of the evaluation of logical arguments, the hierarchy extends indefinitely upward. It is potentially infinite, and therefore maps into the natural number system. The vision we have of the whole, with the upward extending hierarchies, is not fully formalizable. And that means that people are indispensable. In the background of every theory of terms and every theory of arguments there are people. And the people are capable of differing as to whether the terms or arguments they examine match the world in an appropriate way.

9.3 Multiple perspectives

A final contribution from tagmemic theory comes from its affirmation of the possibility of multiple perspectives on the world and on semiotic systems (Pike 1972 [1959], 1982: 19–38). Most of the time, a particular system of formalization offers a single monolithic perspective on the subject-matter that it is supposed to represent. But once we acknowledge openly the role of human participants and analysts, we can raise the question whether the analysts have the choice of more than one perspective through which they may view the topic they consider. What may be the consequence for research topics in analytic philosophy? It may be that a particular area of research can never reach a conclusive endpoint, because it is always possible to approach the whole subject from a new perspective. The multiplicity of perspectives disrupts the feeling of finality.

Moreover, certain applications of tagmemic theory suggest that in many cases the existence of multiple perspectives does not arise *solely* from subjective personal choice. The subject-matter being investigated, whether in language, in music, in logic, or even in physics, may itself innately display multiple dimensions in its structure (Pike 1982: 19–38; Poythress 1976, 1982, 2013a, 2013b, 2015, 2018). This multiplicity again suggests caution when assessing whether the desire for rational mastery can be securely satisfied.

10 Limitations in tagmemic theory?

In all this discussion, I have been using tagmemic theory as the framework of analysis. It remains to ask whether tagmemic theory is subject to the same limitations or objections as those we have seen with respect to symbolic logic and portions of analytic philosophy. Can tagmemic theory be evaluated by a metatheory on a rung above it? Is the whole argument of this present article under permanent suspicion because it has to be re-evaluated in an infinitely ascending hierarchy of evaluations? In one sense, the answer is yes, because it is always possible to generate a hierarchy of analysis and evaluation. But in another sense, the answer is no. Tagmemic theory does not eliminate the personal participant from the theory, but explicitly acknowledges his role. The theory disclaims the ability to capture everything in a formalization. The result is that it is unformalizable, but still insightful.

One of the significant insights is precisely the indispensability of personal human judgment. And that indispensability means also that there is no guarantee that human beings will agree about the application of formalism to the world. In particular, within analytic philosophy, we may expect virtually universal agreement about whether a specific formalized argument conforms to the structure belonging to some particular formalism of symbolic logic. But there is no guarantee that everyone will agree on how or even whether the formalized argument matches substantive aspects of the world or of informal arguments involving human concerns. In the nature of the case, the formal symbolism and the informal analogues never perfectly mirror each other.

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