A Method To Construct Convex, Connected Venn Diagrams for Any Finite Number of Sets

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Given a finite class of sets $A_1, A_2, \ldots, A_n$, a Venn diagram for the class will consist of 2^n regions, each representing a distinct set formed by intersection of the sets and their complements. For example, a typical Venn diagram for three sets A_1, A_2, and A_3 will consist of the eight regions: $A_1 - A_2 - A_3$, $A_2 - A_1 - A_3$, $A_3 - A_1 - A_2$, $A_1 \cap (A_2 - A_3)$, $A_1 \cap (A_3 - A_2)$, $A_2 \cap (A_3 - A_1)$, $A_1 \cap A_2 \cap A_3$, $-A_1 - A_2 - A_3$. The Venn diagram is shown in Figure 1, where A_1 is the upper circle, A_2 is the lower left circle, and A_3 is the lower right circle.

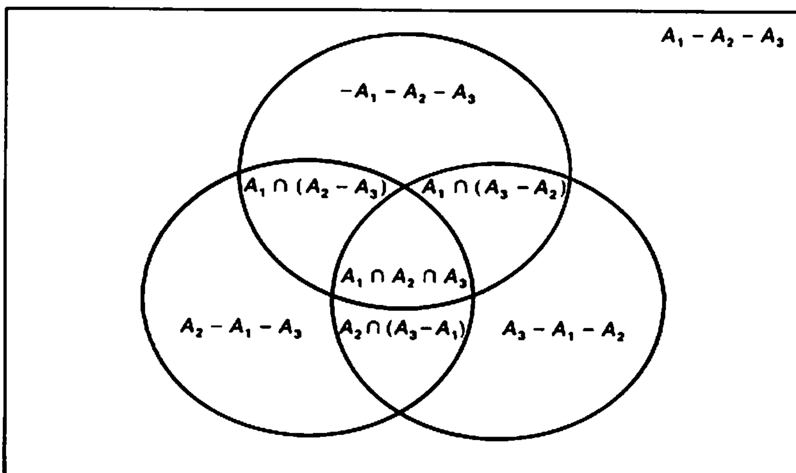

FIGURE 1

Notice that the sets A_1, A_2, and A_3 in Figure 1 are all convex. We can make this a requirement in the construction of a Venn diagram for an arbitrary finite number of sets.

Starting with a square as the universal set, let the upper half be the first set, the right half be the second set, and a circle centered inside the square be the third set. For the rest of the construction, we can concentrate on the upper right quadrant since the same will be done to every quadrant of the square. Divide the right angle formed by the sides of the first and the second sets into 2^{n-2} equal parts. We label the rays $k/2^{n-2}$, $k = 0, 1, 2, \ldots, 2^{n-2}$ in the counter-clockwise manner (for $n = 5$ see Figure 2). Let us call the intersection of the $k/2^{n-2}$-ray

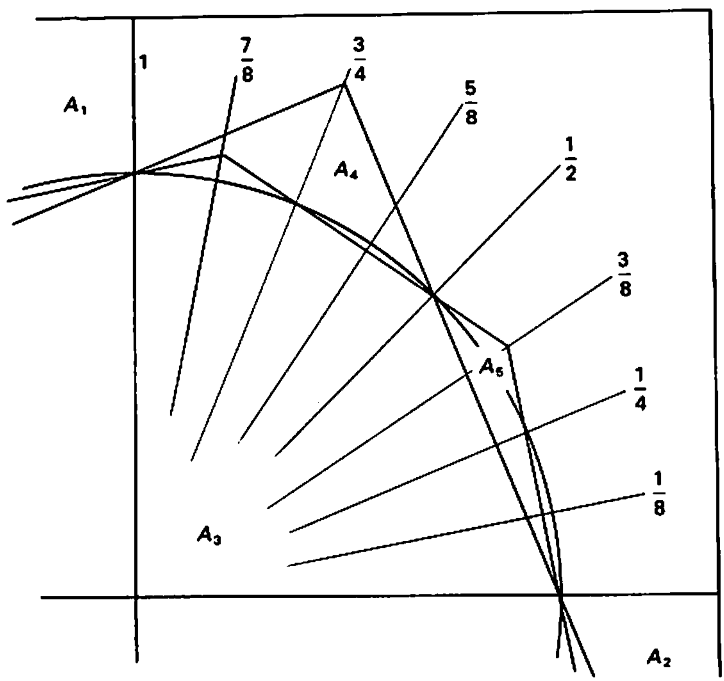

FIGURE 2

and the circle the $k/2^{n-2}$-point for short. The fourth set, A_4, is constructed by drawing a straight line from the 0-point through the 1/2-point, meeting the 3/4-ray at a point, and then joining that point with the 1-point by a straight line that continues into the second quadrant. Notice that the fourth set is a square with one-fourth of it contained in each quadrant of the universal set. The fifth set, A_5, is then an octagon with vertices on the 3/8 and 7/8-rays and sides going through the 0, 1/4, 1/2, 3/4, and 1-points in the upper right quarter.

In general, the set A_k, $k \geq 4$, is a 2^{k-2}-gon, with vertices on the $3/2^{k-2}$, $3 + 4/2^{k-2}$, . . . , and $2^{k-2} - 1/2^{k-2}$, and 1-points. It is easy to see that the set A_k cuts into all the previous regions. Continuing in this way until the set A_n is constructed, we obtain the desired Venn diagram.

To fully appreciate the construction, we conclude with Figure 3, showing the complete construction through A_5.

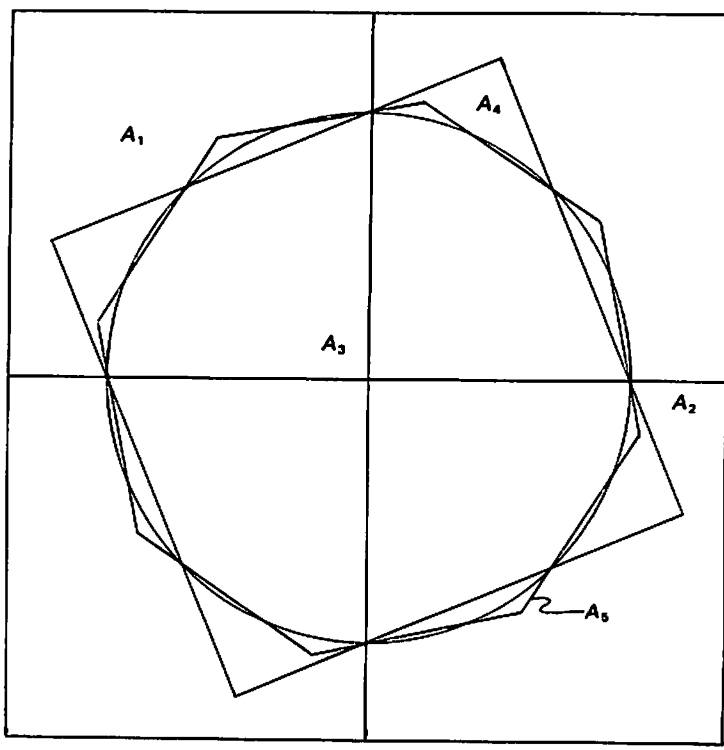

FIGURE 3