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A Formalism for Describing Rules of Conversation

1. INTRODUCTION

In a conversation among several people repeated shifts in the role of 'speaker' and 'addressee' may occur, partly at random (as when a new topic of conversation is started by any one of the participants, who thus becomes the new speaker), partly according to rule (as when a participant responds to a question or comment directed toward him). 'Interruption' or 'rudeness' can then be regarded as a violation of rule. This paper studies the semiotic relation among (a) the social elements of a concrete conversational exchange, (b) the linguistic rules, tacit or explicit, underlying such conversation, and (c) mathematical formalism describing the rules.

The structure of THREE-person conversations has been studied in some detail by Pike (unpublished) from a more mathematical point of view, and by Wise and Lowe (1969) from a more linguistic point of view. I extend Pike's formalism in order to handle rules for n-person conversations, n being an arbitrary positive integer.

2. AN EXAMPLE: A SIMPLE FORMAL BUSINESS MEETING

Rather than proceeding directly to the most general case, I shall introduce the formal techniques in connection with a particular example of ordered conversation, the formal business meeting.

Suppose that such a business meeting involves n participants labelled by the numbers 1, 2, 3,..., n. Let us say that the STATE of the conversation at time t is specified by telling (a) who the chairman is, (b) who the speaker is, and (c) who the addressees are. Thus the state at time t can be represented as a triple (C, S, A) consisting of three subsets $C \subseteq \{1, 2, \ldots, n\}$, $S \subseteq \{1, 2, \ldots, n\}$, and $A \subseteq \{1, 2, \ldots, n\}$. C is a single element subset (such as $\{1\}$) representing the chairman at time t, S is a single element subset representing the speaker at time t, and A is a subset (with one or more elements) representing the set of addressees at time t. If, for example, the chairman 1 is addressing participants 2 and 3, the state is $(\{1\}, \{1\}, \{2, 3\})$.

Besides the state of conversation at any one time, we must consider the rules governing passage from one state to another. As an admittedly oversimplified model of a formal business meeting we can postulate the following rules:

1. The chairman may address the whole group or a single participant.

2. A participant may address only the chairman, and then only when he has been recognized explicitly by the chairman.

3. The chairman acknowledges a participant's response when he resumes the floor. ("Thank you, A., for that remark." "Is that all, A?" etc.)

4. The meeting opens and closes with the chairman addressing the whole group.

A meeting following rules 1-4 can be represented in terms of TRANSFORMATIONS T which result in shifts from one state to another. Let $X = \{1, 2, \ldots, n\}$ be the set of participants. Consider the following transformations.

W, the 'whole group' transformation. If (C, S, A) is any state, let

$$(C,S,A)^{o}W = (C,S,X).$$

W symbolizes a change from the state (C,S,A) to the state (C,S,X). Thus W leaves C and S fixed and changes the set A of addressees to be the whole group X. W is the kind of transformation that occurs when the chairman shifts from addressing a single participant to addressing the whole group. Mathematically, we think of the symbol W as an operator that operates on states (C,S,A) (hence the use of ^{o}) to produce other states $(C', S', A') = (C,S,X)$

Note that the speaker includes himself among the addressees after the transformation has been applied ($S \subseteq X = A'$). We prefer to include the speaker among the addressees when the whole group is being addressed, but the reader may equally well use the transformation W': $(C,S,A)^{o}$

$W' = (C,S,X - S)$ to remedy this anomaly. W' shifts the set of addressees to $X - S$, i.e., the whole group X EXCLUDING the speaker S.

P_k, the 'address k' transformation.[1] If (C,S,A) is any state, let

$$(C,S,A)^{o}P_k = (C,S,\{k\}).$$

P_k leaves the chairman (C) and speaker (S) the same, but the speaker shifts to addressing participant k alone. (For example, "k, what do you think about it?" or "Yes, k?")[2]

R, the 'response' transformation. If (C,S,A) is any state, let

$$(C,S,A)^{o}R = (C,A,S).$$

Thus R symbolizes that the roles of speaker and addressee are interchanged. This kind of transformation occurs when a participant k responds to recognition by the chairman or when the chairman responds to the participant. For example:

Chairman 1: "You have the floor, k." (State ({1}, {1}, {k}).)
Participant k: "Mr. Chairman, I disagree with the remarks of participant j." (State ({1}, {k}, {1}).)
Chairman 1: "Thank you." (State ({1}, {1}, {k}).)

I, the 'identity' transformation. If (C,S,A) is any state, let

$$(C,S,A)^{o}I = (C,S,A).$$

This symbolizes 'no change' in the speaker and addressee roles. One may think of I as the transformation that occurs when a speaker switches to a new subject but roles remain fixed.

The entire course of the idealized business meeting can now be described in terms of a series of transformations. Consider the following meeting.

Chairman 1: "The meeting will come to order." ({1}, {1}, X).
1: "2, please read the minutes." ({1}, {1}, {2}).
2: (reads). ({1}, {2}, {1}).

[1] Later we will include restriction rules on when P_k may occur, to prevent an arbitrary participant from shifting to address participant k.

[2] Such a move by the chairman to 'recognize' participant k almost always presupposes a previous indication from k, either verbal ("Mr. Chairman!", "May I have the floor?", etc.) or kinesic (raising the hand, standing up, etc.). What we have represented as a single shift from, say, ({1}, {1}, X) to ({1}, {1}, {k}) is thus more accurately represented as ({1}, {1}, X) → ({1}, {k}, {1}) (k asking for recognition) → ({1}, {1}, {k}) (participant 1 granting recognition) → ({1}, {k}, {1}) (k speaking to the chair) possibly followed by ({1}, {k}, X). Though the type of formalism we are working with can effectively handle such complexities, we have chosen for illustrative purposes to oversimplify.

1: "Thank you." ({1}, {1}, {2}).
1: "Any corrections?" ({1}, {1}, X).
1: "Any old business?" ({1}, {1}, X).[3]
1: "Any new business?" ({1}, {1}, X).
1: "Yes, 3?" ({1}, {1}, {3}).
3: "Mr. Chairman, about the matter of..." ({1}, {3}, {1}).
1: "O.K., 3." ({1}, {1}, {3}).
1: "Any further discussion?" ({1}, {1}, X).
1: "Meeting adjourned." ({1}, {1}, X).

The sequence of states of the meeting is shown in the left-hand column of Table I, while in the middle and right-hand columns are two sequences of TRANSFORMATIONS representing changes between successive states.

TABLE I

Time	*State*	*Transformation*	*Alternative transformation*
0	({1}, {1}, X)	—	—
1	({1}, {1}, {2})	P_2	P_2
2	({1}, {2}, {1})	R	R
3	({1}, {1}, {2})	R	R
4	({1}, {1}, X)	W	W
5	({1}, {1}, X)	W	I
6	({1}, {1}, X)	W	I
7	({1}, {1}, {3})	P_3	P_3
8	({1}, {3}, {1})	R	R
9	({1}, {1}, {3})	R	R
10	({1}, {1}, X)	W	W
11	({1}, {1}, X)	W	I

Note that the state at times t = 1,2,..., or 11 can be deduced given the initial state s_0 = ({1}, {1}, X) and the series of transformations up to that time; the series of transformations, however, cannot necessarily be deduced from the states. For example, the state ({1}, {1}, {2}) at time 1 is ({1}, {1}, {2}) = ({1}, {1}, X) $\circ$ P_2, the state at time 2 is ({1}, {2}, {1}) = (({1}, {1}, X) $\circ$ P_2) $\circ$ R, the state at time 3 is ({1}, {1}, {2}) = ((({1},

[3] A tacit "no" response to both questions is here presupposed, but omitted in our oversimplified model.

$\{1\}$, X) $\circ$ P_2) $\circ$ R) $\circ$ R, and so on. If we eliminate parentheses and use simple $\circ$ to indicate the composite of two transformations, we can write

$$(\{1\}, \{1\}, \{2\}) = (\{1\}, \{1\}, X) \circ P_2 \circ R \circ R$$

and, for the eleventh time,

$$(\{1\}, \{1\}, X) = (\{1\}, \{1\}, X) \circ P_2 \circ R \circ R \circ W \circ W \circ W \circ P_3 \circ R \circ R \circ W \circ W = S_0 \circ P_2 \circ R \circ R \circ W \circ W \circ W \circ P_3 \circ R \circ R \circ W \circ W.$$

The fact that the transformations I and W have the same effect at some points in the meeting means that we are free to regard either one of these transformations as the one occurring at that point. The meeting as a whole can be represented by either of the two strings

$$S_0\ P_2\ R\ R\ W\ W\ W\ P_3\ R\ R\ W\ W$$
$$S_0\ P_2\ R\ R\ W\ I\ I\ P_3\ R\ R\ W\ I,$$

and the states at any time may be calculated using the strings.

An arbitrary formal business meeting can now be described as an initial state S_0 plus a string of transformations W, P_k, R, with certain restrictions on which transformations can follow which. (We have omitted I from the list because the restriction rules can thereby be simplified.) We postulate the following restrictions on the string of transformations.

(a) $s_0 = (\{k\}, \{k\}, X)$ for some $k = 1, 2, \ldots,$ or n.

This means that one must start with a single chairman k addressing the whole group X (cf. rule 4 given above for a simple formal business meeting).

(b) $s_0 \rightarrow W$

This means that the first transformation symbol of the string, following the state symbol s_0, may be a W. Cf. rule 1.

(c) $s_0 \rightarrow P_k$ for $k = 1, 2, \ldots,$ or n, provided
$\{k\} \neq C$.

This means that any symbol P_k, where $k = 1, 2, 3, \ldots,$ or n, may follow s_0, provided k is not the chairman.

The chairman shifts from addressing the group to addressing k (cf. rule 1).

(d) $W \rightarrow W$

This means that a W may follow a W. No shift occurs in speaker and addressee roles. The chairman may thus continue to address the whole group.

(e) $W \rightarrow P_k$ for $k = 1, 2, \ldots, n$, provided $\{k\} \neq C$.

This means that any symbol P_k, where $k = 1, 2, \ldots$, or n, may follow W, provided k is not the chairman.
This rule is similar to (c).

(f) $P_k \rightarrow W$ for $k = 1, 2, \ldots, n$.

The chairman may readdress the group if k does not respond.

(g) $P_k \rightarrow R$ for $k = 1, \ldots, n$.

Participant k may respond to the chairman (cf. rule 2).

(h) $R \rightarrow R$

The chairman in turn may respond.

(i) When $C = S$, $R \rightarrow W$ and $R \rightarrow P_k$ (for $k = 1, \ldots, n$ but $\{k\} \neq C$).

After the chairman has responded, he may address the whole group ($R \rightarrow W$) or any individual ($R \rightarrow P_k$). However, we allow W and P_k to occur only after an R that leaves the chairman as speaker ($C = S$). Otherwise, we would produce strings like s_0 P_2 R W which allow participant 2 to address the whole group or s_0 P_2 R P_3 which allow participant 2 to address participant 3 directly (both contrary to rule 2).
(i) can be reformulated as:

(i′) After an even number of Rs, $R \rightarrow W$ and $R \rightarrow P_k$ (because an even number of Rs leaves the chairman as the final speaker).

(j) $W \rightarrow \#$

This means that W is a possible symbol preceding the blank symbol # at the end of the string. Cf. rule 4.

Finally, we specify that (a) - (j) are a COMPLETE list of rules, implying that a string for a simple formal business meeting MUST be a string that can be constructed using these rules alone. In particular, such a string must end in a W.

Using rules (a) - (j) the following strings can be produced:

s_0 W
s_0 W W W
s_0 W P_4 W P_5 W W P_2 R R R R P_3 W
s_0 P_4 R R P_2 R R P_2 W P_3 W W

and the following CANNOT be so produced:

s_0 W R
s_0 R W
s_0 P_5 P_4 W
s_0 P_1 R R R W.

In every case it is understood, though not explicitly indicated, that s_0 is subject to rule (a).

The strings such as s_0 W and s_0 W P_4 W P_5 W W P_2 R R R R P_3 W that can be generated using the rules for a particular type of discourse (in this case, a simple formal business meeting) we call COMPLETE CONVERSATIONAL STRINGS (CCS) for that discourse.

To some extent, of course, the rules retain an artificiality, in the sense that variant rules can be imagined. For example, in the case of the simple formal business meeting, we can allow the identity transformation symbol I to be inserted at any point in the middle of a string without seriously affecting the substance of our representation. We would have to add rules

$$\left.\begin{array}{ll} W \rightarrow I & I \rightarrow W \\ P_k \rightarrow I & I \rightarrow P_k \quad (\{k\} \neq C) \\ R \rightarrow I & I \rightarrow R. \end{array}\right\} \text{subject to the restriction that, when the Is are removed, the resulting string conforms to (a) - (j)}$$

Among less trivial changes we can imagine a 'loosening up' of the rules. Let us call a 'simple semiformal business meeting' a conversation subject to the rules (a) - (j) above plus

(m) $P_k \rightarrow P_{k'}$ $\qquad (k, k' = 1, 2, 3, \ldots, n, \quad \{k'\} \neq C)$.

Here the chairman can switch directly from addressing k to addressing k′ without first addressing the whole group (via the sequence P_k W $P_{k'}$). We can, if we wish, add the further liberty that the chairman need not respond, even implicitly, to an address by the k^{th} participant. If we let R_c be the transformation such that $(C, S, A) \circ R_c = (C, C, X)$, this is equivalent to adding the rule

(n) $R \rightarrow R_c$.

3. FORMALISM

With the example of §2 in mind, we now develop a general formalism for describing the course of a conversation.

Let $X = \{1, 2, \ldots, n\}$ be the set of participants in the conversation. Let pX be the set of all subsets of X and $Y = pX - \{\emptyset\}$ the set of all non-empty subsets of X. For example, if $n = 2$, $pX = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$, $Y = \{\{1\}, \{2\}, \{1, 2\}\}$.

In agreement with §2, we say that a STATE of the conversation is a triple (C, S, A), where $C, S, A \in Y$ are non-empty subsets of X. The set of all states is $Y \times Y \times Y = St$. Thus we allow (at least in theory) situations with more than one chairman (when C has more than one element)[4] or more than one speaker (when S has more than one element).[5]

In a discussion group or in an informal conversation among friends there may be no 'chairman' in the sense of a person who possesses special conversational privileges over a period of time; but in a formal business meeting the chairman plays a prominent role. From the point of view of the formalism, we regard a 'chairless' conversation as the limiting case of a 'chaired' conversation in which the chairman can no longer be distinguished from others by any conversational privileges (but cf. the precise definition in §4). In such a chairless conversation, we can assign the role of chairman at random to some one person, who then holds the role throughout the course of the conversation without affecting its structure.

Next, a SHIFT in conversation is a rearrangement of the participants 1, 2,..., n in the roles C, S, A, or, equivalently, a movement $(C, S, A) \rightarrow (C', S', A')$ from one state (C, S, A) at time t to another state (C', S', A') at time $t + 1$.

A TRANSFORMATION T of states is a map
$T: Y \times Y \times Y \rightarrow Y \times Y \times Y$.
We write $(C, S, A) \circ T = (C', S', A')$ to indicate that (C', S', A') is the state resulting from applying the transformation T to the state (C, S, A). W, I, R, etc. of §2 are examples of transformations.

Finally, we can speak of how several transformations are combined in a single conversation as the roles of speaker and addressee continually shift.

We call a string $s_0\ T_1 \ldots T_m$ beginning with a state s_0 followed by transformations T_i a TRANSFORMATIONAL STRING. As in §2, there is asso-

4 Cp. the case of a president and his secretary or the case of a joint chairmanship.
5 Cp. the case of choral recitals or one person speaking 'on behalf of' a caucus.

ciated with a transformational string a corresponding series of states, which is obtained by applying the transformations successively to the starting state. If $s_0 = (C, S, A)$ is the starting state, the STATE STRING $s_0\ s_i \ldots s_m$ associated with the transformational string $s_0\ T_i \ldots T_m$ is defined by $s_1 = s_0 \circ T_1$, $s_2 = s_1 \circ T_2$, ..., $s_i = s_{i-1} \circ T_i$, ..., $s_m = s_{m-i} \circ T_m$, or $s_i = s_0 \circ T_1 \circ \ldots \circ T_i$.

A RULE is a statement (such as (a) – (j) in §2) about the class of transformational strings that can occur in a given type of conversation.

If we denote by $\mathscr{Q}$ the set $(Y \times Y \times Y)^{Y \times Y \times Y} = St^{St}$ of all possible transformations, the set of all possible transformational strings is $St \times \mathscr{Q}^*$. Let us call those transformational strings that are formed according to the rules COMPLETE CONVERSATIONAL STRINGS (CCS). Any initial segment of a ccs is a CONVERSATIONAL STRING (CS). (For example, in the simple formal business meeting, $s_0\ P_4$, $s_0\ P_4\ R\ R$, $s_0\ W\ P_4\ W\ P_5\ WW\ P_2\ R$, as well as $s_0\ W$ and $s_0\ W\ W\ W$ are cs.)

A conversational string that is not a complete conversational string is an INCOMPLETE CONVERSATIONAL STRING (ICS).

4. CLASSES OF CONVERSATION TYPES

For formal purposes, a CONVERSATION TYPE can be identified either with the set of rules about transformational strings or with the set of ccs formed according to the rules. We choose the latter convention. More precisely,

DEFINITION 1. A CONVERSATION TYPE is a set $Q \subseteq St \times \mathscr{Q}^*$ of permissible transformational strings, each string of the form $s_0\ T_1 \ldots T_m$ for some $s_0 \in St$, $m \geqq 0$, and $T_i \in \mathscr{Q}$ for $i = 1, \ldots, m$.

What happens to a conversation type if the participants 1, 2,..., n are permuted? Let $\sigma: X \leftrightarrow X$ be a permutation (1 – 1 onto map) of X into itself. σ induces $p\sigma: pX \leftrightarrow pX$ on the subsets of X, and $(p\sigma)(\emptyset) = \emptyset$. Thus $p\sigma: Y \leftrightarrow Y$ is also a permutation on Y. This in turn induces a map $St(\sigma) = p\sigma \times p\sigma \times p\sigma: Y \times Y \times Y \leftrightarrow Y \times Y \times Y$ component-wise on states $St = Y \times Y \times Y$, and a corresponding permutation σ' of transformations $T: St \rightarrow St$ is defined by

$$s \circ (T^{\sigma'}) = \left(\left(s^{(St(\sigma))^{-1}}\right) \circ T\right)^{St(\sigma)}$$

(all $s \in St$).

The corresponding permutation on transformational strings is σ'':

$$(s_0\ T_1 \ldots T_m)^{\sigma''} = \left(s_0{}^{(St(\sigma))}\right) T_1{}^{\sigma'} \ldots T_m{}^{\sigma'}.$$

Note that $(s_0 \circ T_1 \circ \ldots \circ T_m)^{St(\sigma)} = s_0^{St(\sigma)} \circ T_1^{\sigma'} \circ \ldots \circ T_m^{\sigma'}$, so that the result on state strings is just St (σ). In general, we will abbreviate all the permutations σ, $p\sigma$, St(σ), σ', σ'' as σ, since there is seldom danger of confusion.

The example in §2 showed that the transformation I could be introduced into the formalism (and into ccs) without changing the resulting state strings. Two such conversation types, differing in their ccs but not in their 'output' state strings, will be called STRONGLY SIMILAR. More formally,

DEFINITION 2. Let $Q \subseteq St \times \mathcal{Q}^*$ be the set of ccs of a conversation type. Let State (Q) be the set of state strings associated with the set Q in the natural way:

State (Q) $= \{s_0\, s_1 \ldots s_m : \exists s_0\, T_1 \ldots T_m \in Q$ such that $s_i = s_{i-1} \circ T_i$, $i = 1, \ldots, m\}$.

Two conversation types Q, Q′ are STRONGLY SIMILAR if State (Q) = State (Q′). Q and Q′ are SIMILAR if, for some permutation $\sigma: X \leftrightarrow X$, Q′ is strongly similar to Q^σ.

Informally speaking, Q and Q′ are similar if they produce the same states once the participants have been suitably (by choice of σ) relabelled.

We can now distinguish abstractly several general classes of conversation types.

DEFINITION 3. We say that a conversation type Q is PRIVILEGELESS if, for every permutation $\sigma: X \leftrightarrow X$, Q is strongly similar to Q^σ. Otherwise it is PRIVILEGED.

This means that no one participant or class of participants has conversational advantages over another. A simple formal business meeting is privilegeless, since the formalism itself does not distinguish among the participants 1, 2, ..., n in the sense of affording 'privileges' to some over others at the start.

DEFINITION 4. A conversation type Q is NONHISTORICAL if, for all states s_0, for all m, $p \geq 1$, and for all transformations $T_1, \ldots, T_{m+1}, T_1', \ldots, T_p'$, including a possible blank $T_{m+1} = \#$, the conditions

(i) $s_0 \circ T_1 \circ \ldots \circ T_m = s_0 \circ T_1' \circ \ldots \circ T_p'$

(ii) $s_0\, T_1 \ldots T_m$ is a cs

(iii) $s_0\, T_1' \ldots T_p'$ is a cs

together imply that $s_0\, T_1 \ldots T_m\, T_{m+1}$ is a cs iff $s_0\, T_1' \ldots T_p'\, T_{m+1}$ is a cs.[6]

[6] If $T_{m+1} = \#$, we adopt the convention that '$s_0\, T_1 \ldots T_m \#$ is a cs' means '$s_0\, T_1 \ldots T_m$ is a ccs'. In other words, the statement to be inferred from conditions (i)-(iii) is, in this case, that "$s_0\, T_1 \ldots T_m$ is a ccs iff $s_0\, T_1' \ldots T_p'$ is a ccs".

A conversation type that is not nonhistorical is called HISTORICAL.

In other words, a conversation type is nonhistorical if the possible transformations occurring at any time can be predicted from a knowledge of the preceding state ($s_0 \circ T_1 \circ \dots \circ T_m$) without using 'historical' knowledge extending farther back in time. A conversation type is IMPLICITLY NONHISTORICAL if it is similar to a nonhistorical type.

The simple semiformal business meeting of §2 is non-historical, as can be seen from rewriting the rules entirely in terms of the preceding states:

After state (C, C, {k}), $(C, \{k\}, C) = (C, C, \{k\}) \circ R$, $(C, C, \{k'\}) = (C, C, \{k\}) \circ P_k$, and $(C, C, X) = (C, C, \{k\}) \circ W$ occur.

After state (C, {k}, C), $(C, C, \{k\}) = (C, \{k\}, C) \circ R$ occurs.

After state (C, C, X), $(C, C, X) = (C, C, X) \circ W$, $(C, C, \{k\}) = (C, C, X) \circ P_k$, and # occur,

except that after an initial $s_0 = (\{k\}, \{k\}, X)$ a # cannot occur. This last exception is due to the special character of the initial state. It is provided for in the definition by the condition $m, p \geqq 1$.

DEFINITION 5. We say that states (C, S, A) (C′, S′, A′), are CHAIR-VARIANTS if $S = S'$ and $A = A'$.

DEFINITION 6. A nonhistorical conversation type Q is called CHAIRLESS if, for all states s_0, all $m, p \geqq 1$, and all transformations $T_1, \dots, T_{m+1}, T_1', \dots, T_p'$, including a possible blank $T_{m+1} = \#$, the conditions

(i) $s_0 \circ T_1 \circ \dots \circ T_m = (C, S, A)$ is a chair-variant of $s_0 \circ T_1' \circ \dots \circ T_p' = (C', S, A)$

(ii) $s_0\ T_1 \dots T_m$ is a cs

(iii) $s_0\ T_1' \dots T_p'$ is a cs

together imply that (a) $s_0\ T_1 \dots T_m\ T_{m+1}$ is a cs iff $s_0\ T_1' \dots T_p'\ T_{m+1}$ is a cs, and (b) $s_0 \circ T_1 \circ \dots \circ T_m \circ T_{m+1}$ is a chair-variant of $s_0 \circ T_1' \circ \dots \circ T_p' \circ T_{m+1}$.

Otherwise, a conversation type is CHAIRED.

In informal language, this is to say that information about who fills the (formal) chairman-role C does not affect the subsequent course of conversation. The chair is purely 'honorary'.

5. EXAMPLES

(A) The simple formal business meeting of §2 might be described in brief as follows:

(1) Initial states: ({k}, {k}, X)

Transformations: $(C, S, A) \circ W = (C, S, X)$
$(C, S, A) \circ P_k = (C, S, \{k\})$
$(C, S, A) \circ R = (C, A, S)$

Rules:	$s_0 \rightarrow W$	$P_k \rightarrow W$
	$s_0 \rightarrow P_k$ $(\{k\} \neq C)$	$P_k \rightarrow R$
	$W \rightarrow W$	$R \rightarrow R$
	$W \rightarrow P_k$ $(\{k\} \neq C)$	$R \rightarrow W$ $(C = S)$
	$W \rightarrow \#$	$R \rightarrow P_k$ $(C = S)$

This is a historical privilegeless conversation type (historical because whether or not P_k' can be applied to $(C, C, \{k\})$ depends on whether the chair is initiating or responding to k).

(B) The simple semiformal business meeting of §2 has states, transfor-formations, and rules (1) plus
(2) Rules: $P_k \rightarrow P_k'$ $(\{k'\} \neq C)$.

For further 'loosening' add
(2′) Transformations: $(C, S, A) \circ R_c = (C, C, X)$

Rules: $R \rightarrow R_c$.

This conversation type, with or without (2′), is a nonhistorical chaired privilegeless conversation type.

(C) A SIMPLE INFORMAL BUSINESS MEETING is the conversation type with states, transformations, and rules (1), (2), and (2′) plus
Transformations: $(C, S, A) \circ B_k = (C, \{k\}, C)$

Rules: $W \rightarrow B_k$ $(\{k\} \neq C)$.

This permits a participant to respond to the chair without being explicitly acknowledged (via W P_k R); however, k cannot 'butt in' when k′ is being addressed (e.g., the sequence P_k, B_k is not permitted). This type is non-historical chaired privilegeless.

We can also deal with a more complex business meeting where shifts of chairman are possible.

(D) A COMPLEX FORMAL BUSINESS MEETING has Initial states: $(\{k\}, \{k\}, X)$

Transformations: W, R, and P_k of (1) and $(C, S, A) \circ Pc_k = (\{k\}, S, A)$ (k becomes chair, leaving S and A the same).

Rules: the rules of (1) plus

$W \rightarrow P_{c_k}$ (the chairman announces to the meeting that k is the new chairman)

$P_{c_k} \rightarrow P_k$ (the chairman turns the meeting over to k: "k, the meeting is yours.")

plus the prohibition that the sequence $P_{c_k}\ P_k\ W$ is not allowed (the new chairman MUST take over by $P_{c_k}\ P_k\ R$). This type is historical privilegeless.

(E) As another alternative, a COMPLEX FORMAL BUSINESS MEETING WITH SUBORDINATE CHAIRS, consider the case where the chairman is permanent but 'subordinate chairs' can be appointed during the course of the meeting to handle the business of a certain portion of the meeting. This type of meeting can be described in exactly the same way as the complex formal business meeting (D), with the extra rule that the chair must eventually revert to the original chairman by the same route it moved away from him. More formally, we want that, if (i) all the transformations except the P_{c_k}'s are deleted from a ccs, (ii) all sequences of the form ... $P_{c_{k_1}} P_{c_{k_2}} P_{ck_1}$ ... are changed to ... $P_{c_{k_1}}$ the final result is either s_0 or $s_0\ P_{c_{k_1}}\ P_{c_{k_0}}$, where $s_0 = (\{k_0\}, \{k_0\}, X)$.
For example, $s_0\ W\ P_{k_1}\ R\ R\ P_{k_2}\ W$ reduces by (i) to s_0; $s_0\ W\ P_{c_{k_1}}\ P_{k_1}\ R\ W\ P_{c_{k_0}}\ P_{k_0}\ R\ W$ reduces to $s_0\ P_{c_{k_1}}\ P_{c_{k_0}}$ by (i); and

$s_0\ W\ P_{c_{k_1}}\ P_{k_1}\ R\ W\ P_{c_{k_2}}\ P_{k_2}\ R\ W\ P_{c_{k_3}}\ P_{k_3}\ R\ W\ P_{c_{k_2}}\ P_{k_2}\ R\ W\ P_{c_{k_4}}\ P_{k_4}\ R\ W\ P_{c_{k_2}}\ P_{k_2}\ R\ W\ P_{c_{k_1}}\ P_{k_1}\ R\ W\ P_{c_{k_0}}\ P_{k_0}\ R\ W$ reduces to

$s_0\ P_{c_{k_1}}\ P_{c_{k_2}}\ P_{c_{k_3}}\ P_{c_{k_2}}\ P_{c_{k_4}} P_{c_{k_2}}\ P_{c_{k_1}}\ P_{c_{k_0}}$ by (i), thence to

$s_0\ P_{c_{k_1}}\ P_{c_{k_2}}\ P_{c_{k_4}}\ P_{c_{k_2}}\ P_{c_{k_1}}\ P_{c_{k_0}}$ to $s_0\ P_{c_{k_1}}\ P_{c_{k_2}} P_{c_{k_1}}\ P_{c_{k_0}}$ to $s_0\ P_{kc_1}\ P_{c_{k_0}}$ by (ii).

On the other hand, $s_0\ W\ P_{c_{k_1}}\ P_{k_1}\ R\ W$ is not acceptable because it does not so reduce. This conversation type is a good example of a highly historical privilegeless conversation type, in which information from previous states needs to be retained indefinitely long.

(F) In a similar manner we may construct COMPLEX SEMIFORMAL and INFORMAL BUSINESS MEETINGS, with or without 'subordinate chairmen.

Now we turn to various types of 'discussion groups', in which all the participants can address the whole group.

(G) A SIMPLE LEADERLESS DISCUSSION GROUP has Initial states: $s_0 = (X, \{k_0\}, X)$

Transformations: $(C, S, A) \circ B_k = (C, k, C)$

Rules: $s_0 \rightarrow B_k \ (k = 1, 2, \ldots, n)$
$B_k \rightarrow B_k'$
$B_k \rightarrow \#.$

This is a nonhistorical chairless privilegeless conversation type. The ccs are of the form

$$s_0 \, B_{k_1} \, B_{k_2} \ldots B_{k_m},$$

from which it can be immediately read off that $k_0, k_1, k_2, \ldots, k_m$ successively spoke to the whole group, after which the discussion ended.

(H) A SIMPLE LEADERLESS DISCUSSION GROUP WITH OPENER AND CLOSER can be constructed by specifying that a given person, say 1, must open and close:

$s_0 = (X, \{1\}, X)$
$s_0 \rightarrow B_k$
$B_k \rightarrow B_k'$
$B_1 \rightarrow \#$

or that the same person must open and close:

$s_0 = (X, \{k_0\}, X)$ for any $k_0 = 1, 2, \ldots,$ or n
$s_0 \rightarrow B_k$ $\qquad B_k \rightarrow B_k'$
$B_{k_1} \rightarrow \#$ only if $k_1 = k_0$.

The former of the two is a nonhistorical chairless privileged conversation type, while the latter is historical privilegeless.

(I) In a LEADERLESS DISCUSSION GROUP WITH QUESTIONS one might allow the participants to direct questions to any individual as well as to address the whole group.

$s_0 = (X, k, X)$ for any $k = 1, \ldots,$ or n.
$s_0 \rightarrow B_k$
$B_k \rightarrow B_k'$
$B_k \rightarrow P_k'$ (k now addresses k′)
$P_k' \rightarrow R$ (k′ may respond)
$P_k' \rightarrow W$ (k′ may not respond; then k readdresses the group)
$R \rightarrow R$ $\qquad W \rightarrow B_k$ $\qquad W \rightarrow \#$
$R \rightarrow W$ $\qquad W \rightarrow P_k$ $\qquad B_k \rightarrow \#$

(J) One may have a LED DISCUSSION GROUP in which only the leader, participant 1, may ask questions of individuals.

$s_0 = (X, \{1\}, X)$
$s_0 \rightarrow B_k$
$s_0 \rightarrow P_k$ (for $k \neq 1$) (the leader asks a question or makes a comment to k)
$B_k \rightarrow B_k'$
$B_1 \rightarrow P_k$ (for $k \neq 1$) $\quad R \rightarrow P_k$
$P_k \rightarrow R \quad R \rightarrow B_k$
$P_k \rightarrow B_1 \quad B_1 \rightarrow \#$

This is a nonhistorical chairless privileged conversation type.

(K) One may have INDIVIDUALISTIC DIALOGUE where any participant may address only a single individual at a time.
$s_0 = (X, \{k_1\}, \{k_2\}) \quad (k_1 \neq k_2)$.
$s_0 \rightarrow P_{k_2}$
$P_{k_1} \rightarrow P_{k_2}$ if k_2 is not the speaker
$R \rightarrow P_{k_2} \quad R \rightarrow \#$
$R \rightarrow R \quad P_k \rightarrow \#$

The number of other conversation types that could be listed is limited by little but the reader's imagination. These examples, however, should suffice as illustrations of how the formalism can be put to work.

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